Rethinking the Geometry of the Demand and Supply Functions

Saidou Baba Oumar[1]

Abstract

This paper attempts to revisit the geometry of the demand and supply functions in economic analysis, taking mathematical exigencies into account. Leaning on the fundamental principles that guide the theory of demand and supply, the paper uses the analytical approach of data analysis to address the objectives of the investigation. Results of the study indicate that whether expressed in the standard mathematical form (Quantity-price relation) or casual mathematical form (Price-quantity relation), the demand and supply functions obey the laws of demand and supply. Also, the results reveal that the first component in the computation of the price-elasticity remains unchanged and takes into account the quantity-price relationship, irrespective of the functional form (Quantity-price function or price-quantity function) used in defining the demand and supply functions. Furthermore, the results show that the consumer and producer surpluses can still be evaluated without violating mathematical requirements with the quantity-price relation model of describing the demand and supply functions to accommodate welfare issues associated with the demand and supply concepts. As a result, it is suggested that mathematical requirements of logic, rigor, consistency, and objectivity be strictly respected when subjecting the analysis of real world economic phenomena to mathematical treatments.

Keyword: Consumer, Economics, Elasticity, Mathematics, Price, Producer, Quantity, Surplus

JEL Classifications: D01, D11, D20, D60, I30

1. Introduction

The use of demand and supply concepts, though vaguely described at the origin, dates back to the 14th century with ibn Taimiyah – a Muslim Syrian scholar who argued that when the desire for a good or service augments and its availability shrinks, its price rises. Conversely, when the availability of the good increases and the desire for it diminishes, its price falls (ibn Taimiyah 1381: 523 as cited in Islahi 1983: 51). At the dusk of the 17th century, Davenant (1699: 224-226) as cited in

[1] Department of Economics, The University of Bamenda, Cameroon, Department of Economics, University of Buea, Cameroon, email: saidoubo@gmail.com

Endres (1987: 621-623) disseminated an explanation of the relationship between the price and harvested quantity of corn in England, initially offered by King (1696) as cited in Barnett (1936) that if the harvest of corn drops by half, the price augments by 500%. Without clearly mentioning the terms demand and supply, Locke (1691) introduced an unambiguous description of the relationship between the variables involved in the concepts, claiming that the rise or fall in the price of any commodity depends on the size of the market (Number of buyers and sellers), and the quantity readily available for the market. It is Mill (1767) who first mentioned explicitly the phrase "demand and supply" in economic discussions which offered a greater opportunity for its use among economists of his time such as Smith (1776) as regards the invisible hand that naturally guides the course of action of economic agents – consumers or producers in an economy, and Ricardo (1817) with reference to the economic rent. Yet, it is necessary to recall that the first mathematical model of demand and supply originated from Cournot (1838). The graphic representation of demand and supply is credited to Jenkin (1870), while its expansion and publicity is owed to Marshall (1890) who in his inquiries reconciled the demand and supply side explanations of value into a single analytical construction – the scissors diagram.

Economic analyses are governed by the forces of demand and supply, which are sometimes referred to as market mechanisms. In trying to apprehend economic phenomena, economists depend on statistics and mathematics in addition to economic theories. In the fields of statistics and mathematics, analytical tools such as probability, algebra, calculus, logic, sets, and geometry are used to describe and explain real world behaviors of economic agents. Depending on the purpose of analysis, the economic phenomena under investigation are often captured in algebraic expressions with the help of equations and functions or illustrated geometrically with the aid of graphs and curves.

The primary role of a function is to construct a model that establishes a mathematical or physical relationship between a set of dependent variable and another set of independent or explanatory variable(s). Generally, the functional relation between the two sets of variables is expressed as in equation (1), where Y is the dependent variable and x_i is the explanatory variable for $i = 1,2,3,\ldots n$.

$$Y = f(x_i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

In the case of the demand for and supply of a good or service in a competitive market, the equivalent functions can be specified once the determinants of each phenomenon are identified. As a result, the demand and supply functions can be expressed as in equations (2) and (3), respectively.

$D_0 = f(P, P_c, P_g, I, T, W, C, E, A, P_o .....) \ldots$(2), where P is own-price of the good, P_c represents price of complement goods, P_g is price of substitute goods, I is income level, T stands for tastes and preferences, W represents weather condition, C corresponds to customs and believes, E captures expectations, A is age of the consumer, and P_o stands for population.

$S_0 = f(P, F, H, W, G, E, A, P_o .....) \ldots\ldots\ldots$ (3), where F is price of inputs and H represents level of technology. The meaning of the other determinants remains the as explained in equation (2).

Holding all other determinants constant and taking the own-price of the good as the most important influencing factor, the short-run demand and supply functions in equations (2) and (3) can be reduced to functions of one dependent variable with one explanatory variable. Consequently, equation (2) translates into equation (4) and equation (3) becomes equation (5), where P is own-price of the good.

$D_0 = f(P) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (4)
$S_0 = f(P) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (5)

In equations (4) and (5) the quantities demanded for and supplied of the good depend on the market price (P) of the good (Quantity-price relation). This is the standard mathematical model of the demand and supply functions of a good. Alternatively, equations (4) and (5) can be expressed as functions of P, where Q is the explanatory variable (Price-quantity relation). The latter form is considered as the casual mathematical model of the demand and supply functions of a good, respectively represented in equations (6) and (7).

$P_d = f(Q) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (6)
$P_s = f(Q) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (7)

Yet it is necessary to note that although Q or P can be used as the dependent variable to model the demand and supply functions of a good, this flexibility does not apply between Q and the other determinants of demand and supply. For instance, the quantity demanded for and supplied of a good is often influenced by the weather condition (*W*) or population (P_o), but the reverse is not true. Besides, irrespective of the option adopted in modeling the demand and supply functions, the geometry of the model has to conform to the mathematical requirement of representing a function graphically on a Cartesian plane diagram. This implies the plotting of the dependent variable on the vertical axis and that of the explanatory variable on the horizontal axis. Humphrey (1992) explains that before Marshall (1890), economists such as Cournot (1838), Rau (1841), Dupuit (1844), von Mangoldt (1863), and Jenkin (1870) complied with this mathematical provision of geometry by representing the dependent variable (*Q*) of the standard mathematical model of the demand and supply functions of a good on the vertical axis and their explanatory variable (*P*) on the horizontal axis of the Cartesian plane diagram. Some years later the practice changed with Alfred Marshall. As a result of the new dispensation, a series of questions arises. Why did Marshall (1890) interchange the position of the variables on the axes by plotting the dependent variable (*Q*) on the horizontal axis and the explanatory variable (*P*) on the vertical axis of the Cartesian plane diagram? Is this pattern of transposing the axes of the variables common to all graphic analyses and representations of real world economic phenomena? Are there sufficient arguments to motivate the graphic representation of the demand and supply functions with the dependent variable (*Q*) plotted on the vertical axis and the explanatory variable (*P*) on the horizontal axis of the Cartesian plane diagram again?

Based on this series of questions, the paper attempts to revisit the geometry of the demand and supply functions in economic analysis as conceived by Marshall (1890). Specifically, the paper intends:

- To prove the validity of the laws of demand and supply with regard to the two mathematical forms of defining the demand and supply functions of a good,
- To examine the procedure for computing the price-elasticity of demand and supply,
- To explore the computation of consumer and producer surpluses,
- To discuss the geometry of other economic analyses, and
- To recommend the appropriate ways of using mathematics in economic analysis.

The rest of the paper is organized as follows. Section 2 discusses the theoretical framework, where emphasis is placed on the theory of demand and supply. Section 3 explains the methodology adopted to capture the objectives of the inquiry. Section 4 presents the analysis and discussion of data with numerical proofs and graphic representations of the investigated phenomenon. Section 5 concludes the study and offers recommendations on how to capture the investigated phenomenon graphically in consonance with mathematical requirements.

2. Theoretical Framework

The investigation carried out in this paper leans on the theory of demand and supply. Two fundamental principles guide this theory. These are the law of demand and law of supply.

The law of demand claims that in a normal demand function, the quantity demanded for a good relates inversely with the price of the good being consumed by the economic agent. The inverse or negative relationship between the price and quantity demanded for the good suggests that at higher price levels, lesser quantities of the good are acquired by the consumer and vice versa. This description is generally referred to as the first law of demand. The law of supply advocates a direct or positive relationship between the price and the quantity supplied of a good in a normal supply function, implying that at higher price levels, higher quantities of the good are supplied by the producer and at lower price levels, lesser quantities of the good are offered to the consumer. This narrative is generally referred to as the first law of supply.

Economic theory suggests a specific relationship between the dependent and explanatory variables of the demand and supply functions. This relationship is guided by the principles of demand and supply. The law of demand insists on a negative (Inverse or indirect) relationship between the dependent and explanatory variables of the demand function, while the law of supply claims a positive (Direct) relationship between the dependent and explanatory variables of the supply function. Going by the principles of demand and supply, and considering the standard mathematical way of describing the demand and supply functions of a good, equations (4) and (5) can explicitly be written as shown in equations (8) and (9), respectively where D_0 and S_0 are the dependent variables and P is the explanatory variable.

$$D_0 = f(P) = \alpha_1 - \beta_1 P \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$
$$S_0 = f(P) = \alpha_2 + \beta_2 P \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Based on the laws of demand and supply, and focusing on the casual mathematical way of defining the demand and supply functions of a good, equations (4) and (5) can also be expressed explicitly as indicated in equations (10) and (11), respectively. In these equations, P_d and P_s represent the dependent variables and Q is the explanatory variable.

$$P_d = f(Q) = \alpha_3 - \beta_3 Q \text{ .... } (10)$$
$$P_s = f(Q) = \alpha_4 + \beta_4 Q \text{ .... } (11)$$

In equations (8) and (10), the expected inverse relationship between the quantity demanded for a good and its market price is fulfilled because when Q is preceded with a positive sign (+), P is affected by a negative sign (–) and vice versa. Similarly, the anticipated direct relationship between the quantity supplied of a good and its market price is satisfied in equations (9) and (11) because both Q and P are affected by a positive sign (+).

3. Methodology

This paper adopts the analytical approach of data analysis to achieve the objectives of the inquiry. A comparison of the graphic representation of several other functions used in describing economic real world occurrences with the geometry of the demand and supply functions is carried out regarding the plotting of the dependent and explanatory variables on the Cartesian plane diagram. Furthermore, the computational procedure of the price elasticity of demand and supply is considered and confronted with the two available options of expressing the demand and supply functions of a good to challenge the classical approach of representing and illustrating the geometry of the demand and supply functions, thereby justifying the approach proposed in this paper. Also, numerical data are applied on the demand and supply functions models to verify the validity of the laws of demand and supply as regards the standard and casual mathematical forms of defining the demand and supply functions of a good, accompanied with graphic representations. In addition a procedure of evaluating the consumer and producer surpluses is suggested to accommodate welfare issues associated with the demand and supply concepts. The paper heavily depends on secondary data to perform the analysis.

4. Data Analysis and Discussion

The demand and supply functions correspond to an economic model of price-quantity determination in a market. In a free market, the forces of demand and supply interact to clear the market and make it settle at a position where the quantity demanded for at the prevailing price equals the quantity supplied at that price, thus resulting to an economic equilibrium position (E_Q) for the agreed price and quantity. It is worth mentioning that wherever units of measurement are needed in the analysis, the price is quoted in *Franc de la Communauté Financière de l'Afrique* (FCFA) and the quantity is measured in Kilograms (Kg).

4.1 Relationship between Variables of the Demand and Supply Functions

To verify the validity of the laws of demand and supply with regard to the standard and casual mathematical forms of defining the demand and supply functions of a good, numerical examples are applied. The verification is firstly done for the standard mathematical form and secondly undertaken for the casual mathematical form with corresponding numerical examples.

4.1.1 Verification with the Standard Mathematical Form

The standard mathematical form of presenting the demand and supply functions advocates the dependence of the quantity (Demanded and supplied) on the price of the good in question. Equations (12) and (13) are respective examples.

$$D_0 = f(P) = 50 - 2P \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (12)$$
$$S_0 = f(P) = 20 + 3P \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (13)$$

Equating equation (12) to equation (13) yields an equilibrium price of $P = 6$. Substituting the equilibrium price into equations (12) and (13) gives an equilibrium quantity of $D_0 = S_0 = 38$. Two scenarios are considered in the verification of the principles of demand and supply with the standard mathematical form expressed in equations (12) and (13).

Scenario 1: Using arbitrary values of P = 0, 1, 2, 3… 10 and substituting them into equation (12) and equation (13) generates the various quantities of the good that can be acquired or offered for sale in the market. A comparison of the first row and second row of Table 1 shows that as the price of the good increases from one level to the next, the quantity demanded for the good decreases and vice versa. This confirms the law of demand. Similarly, a comparison between the first and third rows of Table 1 explains that as the price of the good rises from one level to the next, the quantity supplied of the good also rises and as the price of the good drops from one level to the next, the quantity supplied also follows the same trend. This substantiates the law of supply.

Table 1. Quantity-price schedules of demand for and supply of a good in a free market (Scenario 1)

Above or below equilibrium position							*EQ*	Below or above equilibrium position			
Price (P)	0	1	2	3	4	5	***6***	7	8	9	10
Demand (D_0)	50	48	46	44	42	40	***38***	36	34	32	30
							38				
Supply (S_0)	20	23	26	29	32	35		41	44	47	50

A graphic representation of equations (12) and (13) is illustrated in Figure 1. The dependent variable–quantity (Q) is plotted on the vertical axis and the explanatory variable– price (P) appears on the horizontal axis. The demand function with its negative slope is represented by D_0, and the supply function with its positive slope is identified by S_0. The equilibrium position of the market is established where the two equations meet at unique values of $D_0 = S_0 = 38$ for the quantity demanded for and supplied of the good in the market and $P = 6$ for the market price of the good.

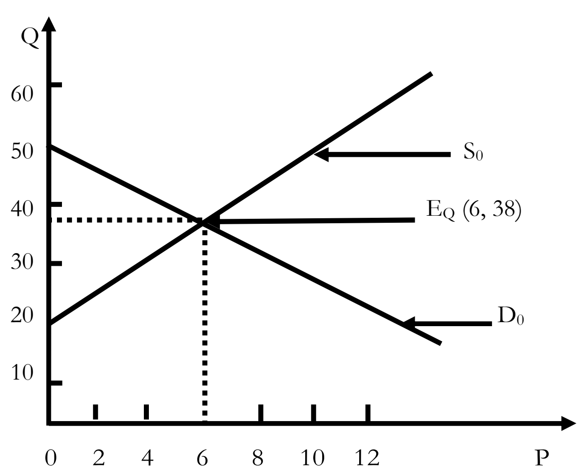

Figure 1. Quantity-price of demand and supply curves of a good in a free market (Scenario 1)

Scenario 2: Using arbitrary values of Q = 0, 1, 2, 3… 54 and substituting them into equation (12) and equation (13) generates the various prices of the good that can apply in the market. A comparison of the row for quantity (Q) with the row for price (P_d) of Table 2 shows that as the quantity demanded for the good rises from one level to the next, the price of the good decreases and vice versa. This validates the law of demand. Also, a comparison between the row for quantity (Q) and the row for price (P_s) of Table 2 explains that as the quantity supplied of the good increases from one level to the next, the price of the good also increases and as the quantity supplied of the good decreases from one level to the next, the price of the good declines. This verifies the law of supply.

Table 2. Price-quantity schedules of demand for and supply of a good in a free market (Scenario 2)

Quantity (Q)	0	1	2	3	4	5	6	7	8	9	10
Demand (P_d)	25	24.5	24	23.5	23	22.5	22	21.5	21	20.5	20
Supply (P_s)	-6.66	-6.33	-6	-5.66	-5.33	-5	-4.66	-4.33	-4	-3.66	-3.33
Quantity (Q)	11	12	13	14	15	16	17	18	19	20	21
Demand (P_d)	19.5	19	18.5	18	17.5	17	16.5	16	15.5	15	14.5
Supply (P_s)	-3	-2.66	-2.33	-2	-1.66	-1.33	-1	-0.66	-0.33	0	0.33
Quantity (Q)	22	23	24	25	26	27	28	29	30	31	32
Demand (P_d)	14	13.5	13	12.5	12	11.5	11	10.5	10	9.5	9
Supply (P_s)	0.66	1	1.33	1.66	2	2.33	2.66	3	3.33	3.66	4
Quantity (Q)	33	34	35	36	37	***38***	39	40	42	43	44
Demand (P_d)	8.5	8	7.5	7	6.5	***6***	5.5	5	4.5	4	3.5
Supply (P_s)	4.33	4.66	5	5.33	5.66	***6***	6.33	6.66	7	7.33	7.66
Quantity (Q)	45	46	47	48	49	***EQ***	50	51	52	53	54
Demand (P_d)	3	2.5	2	1.5	1	-	0.5	0	-0.5	-1	-1.5
Supply (P_s)	8	8.33	8.66	9	9.33		9.66	10	10.33	10.66	11
Above or below equilibrium position							Below or above equilibrium position				

A graphic illustration of equations (12) and (13) is given in Figure 2. The dependent variable–quantity (Q) is captured on the horizontal axis and the explanatory variable–price (P) is represented on the vertical axis. The demand function with its negative slope is represented by P_d, and the supply function with its positive slope is identified by P_s. The equilibrium position of the market is achieved where the two equations meet at unique values of $Q = 38$ for the quantity demanded for and supplied of the good in the market and $P_d = P_s = 6$ for the market price of the good. Note that under this scenario the graphic illustration of the demand and supply functions discarded the values of Q ranging from 0 to 19 due to the negative price values that affected the supply function P_s, which therefore contradicts economic theory. The supply function P_s starts taking positive values from Q = 20 upward.

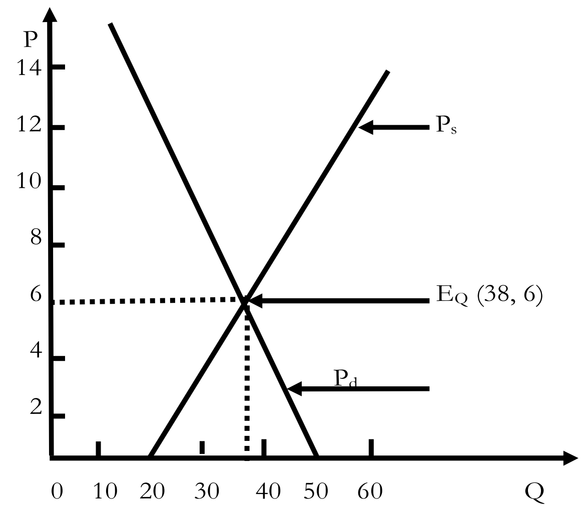

Figure 2. Price-quantity of demand and supply curves of a good in a free market (Scenario 2)

4.4.2 Verification with the Casual Mathematical Form

Using equations (12) and (13) and making P the subject of the demand and supply equations yields their respective demand and supply functions as captured in equations (14) and (15). To verify the applicability of the laws of demand and supply with the casual mathematical form expressed in equations (14) and (15), two scenarios are once more taken into account.

$$P_d = f(Q) = 25 - 0.5Q \quad \text{.....................} \quad (14)$$

$$P_s = f(Q) = \frac{1}{3}Q - \frac{20}{3} \quad \text{......................} \quad (15)$$

Scenario 1a: Using arbitrary values of Q = 0, 1, 2, 3… 54 and substituting them into equation (14) and equation (15) generates the various prices of the good that can apply in the market as obtained in Scenario 2 of the verification with the standard mathematical form of expressing the demand and supply functions presented in equations (12) and (13). Table 3 displays the corresponding price values of the good in the market.

Table 3. Price-quantity schedules of demand for and supply of a good in a free market (Scenario 1a)

Quantity (Q)	0	1	2	3	4	5	6	7	8	9	10
Demand (P_d)	25	24.5	24	23.5	23	22.5	22	21.5	21	20.5	20
Supply (P_s)	-6.66	-6.33	-6	-5.66	-5.33	-5	-4.66	-4.33	-4	-3.66	-3.33
Quantity (Q)	11	12	13	14	15	16	17	18	19	20	21
Demand (P_d)	19.5	19	18.5	18	17.5	17	16.5	16	15.5	15	14.5
Supply (P_s)	-3	-2.66	-2.33	-2	-1.66	-1.33	-1	-0.66	-0.33	0	0.33
Quantity (Q)	22	23	24	25	26	27	28	29	30	31	32
Demand (P_d)	14	13.5	13	12.5	12	11.5	11	10.5	10	9.5	9
Supply (P_s)	0.66	1	1.33	1.66	2	2.33	2.66	3	3.33	3.66	4
Quantity (Q)	33	34	35	36	37	***38***	39	40	42	43	44
Demand (P_d)	8.5	8	7.5	7	6.5	***6***	5.5	5	4.5	4	3.5
Supply (P_s)	4.33	4.66	5	5.33	5.66	***6***	6.33	6.66	7	7.33	7.66
Quantity (Q)	45	46	47	48	49	***EQ***	50	51	52	53	54
Demand (P_d)	3	2.5	2	1.5	1	**-**	0.5	0	-0.5	-1	-1.5
Supply (P_s)	8	8.33	8.66	9	9.33		9.66	10	10.33	10.66	11
Above or below equilibrium position							Below or above equilibrium position				

The corresponding diagram for equations (14) and (15) is given in Figure 3, with the dependent variable–price (P) captured on the vertical axis and the explanatory variable–quantity (Q) placed on the horizontal axis. The demand function is represented P_d, and the supply function is described by P_s. The equilibrium position of the market is attained where the two equations meet at unique values of $Q = 38$ for the quantity demanded for and supplied of the good in the market and $P_d = P_s = 6$ for the market price of the good. Again, the negative price values that affected the supply function P_s were excluded from the graphic design of the demand and supply functions for Q ranging from 0 to 19 because of the contradiction with economic theory. It is once more observed that the supply function P_s starts taking positive values only from $Q = 20$ upward.

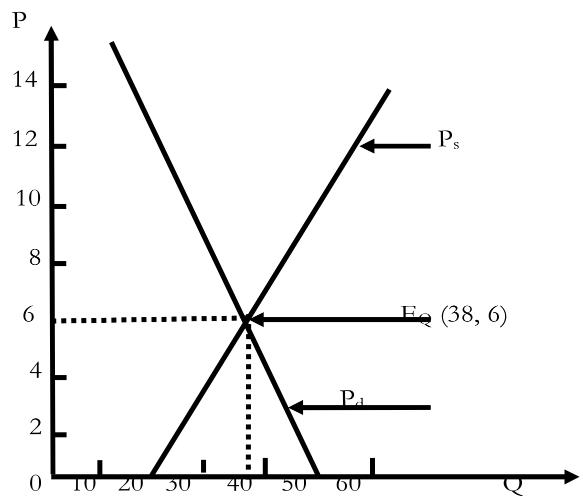

Figure 3. Price-quantity of demand and supply curves of a good in a free market (Scenario 1a)

Scenario 2a: Using arbitrary values of P = 0, 1, 2, 3… 10 and substituting them into equation (14) and equation (15) generates the various quantities of the good that can be acquired or offered for sale in the market. A comparison of Row 1 with Row 2 of Table 4 validates the law of demand, while a confrontation of Row 1 with Row 3 of Table 4 confirms the law of supply.

Table 4. Quantity-price schedules of demand for and supply of a good in a free market (Scenario 2a)

	Above or below equilibrium position						***EQ***	Below or above equilibrium position			
Price (P)	0	1	2	3	4	5	***6***	7	8	9	10
Demand (D_0)	50	48	46	44	42	40	***38***	36	34	32	30
Supply (S_0)	20	23	26	29	32	35	***38***	41	44	47	50

Figure 4 depicts the graphic illustration of equations (14) and (15). The dependent variable-quantity (Q) is plotted on the vertical axis and the explanatory variable–price (P) appears on the horizontal axis. The demand function is identified by D_0, and the supply function is represented by S_0. The equilibrium position of the market is reached where the two equations meet at unique values of $D_0 = S_0 = 38$ for the quantity demanded for and supplied of the good in the market and $P = 6$ for the market price of the good as obtained in Scenario 1 of the verification of the laws of demand and supply with the standard mathematical form of expressing the demand and supply functions.

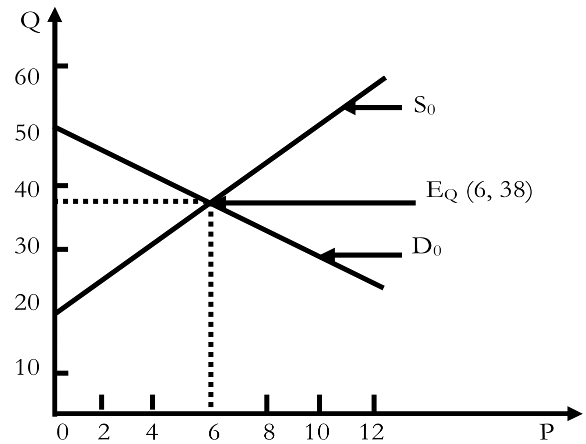

Figure 4. Quantity-price of demand and supply curves of a good in a free market (Scenario 2a)

Depending on whether the demand and supply functions are modeled with the standard or casual mathematical form, the laws of demand and supply always hold. The inverse relationship between the price and quantity demanded for a good is always maintained for the demand function, and the direct relationship between the price and quantity supplied of a good is always satisfied for the supply function. However, it is observed that the standard mathematical form (*Q* depending on *P*) is more responsive to mathematical treatments than the casual mathematical form (*P* depending on *Q*) on the following grounds.

One, the requirement of objectivity in the retention of values computed from the demand and supply functions as presented is respected. Since the observed demand and supply functions describe an economic model of price and quantity determination in a specific market at a given time, all computed values of the dependent variable (*Q* for standard mathematical form, and *P* for casual mathematical form) obtained from selected interval values of the explanatory variable (*P* for standard mathematical form, and *Q* for casual mathematical form) of the two functions must be retained for the graphic construction of the model. This requirement is only fulfilled with the standard mathematical form as presented in Figure 1 and Figure 4. The casual mathematical form of modeling the demand and supply functions exclude some computed values of the dependent variable in the graphic representation of the model, especially the negative supply values for interval values of the explanatory variable below $Q = 20$ as illustrated in Figure 2 and Figure 3. Two, the computation of corresponding values of the dependent variable for a given interval values of the explanatory variable is easier with the standard mathematical form as presented in Table 1 and Table 4 than with the casual mathematical form as shown in Table 2 and Table 3 of modeling the demand and supply functions.

4.2 The Price-elasticity of Demand and Supply Functions

The computational procedure of the price-elasticity of demand and supply is done generally by taking the ratio of the percentage change in quantity demanded or supplied to the percentage change in the price of the good. This is mathematically captured in equation (16), where E_p is the price-elasticity; $\%\Delta Q$ stands for percentage change in quantity and $\%\Delta P$ represents percentage change in price as conceived by Marshall (1890).

$$E_p = \frac{\%\Delta Q}{\%\Delta P} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(16)}$$

Equation (16) can be decomposed into two components and expressed as in equation (17), where the first component accounts for the ratio of change in quantity to change in price and the second term for the ratio of price to quantity. Recall that for point elasticity, the computational procedure considers either the initial or end values of the selected points in the second term of equation (17), whereas the computational procedure for arc elasticity considers the average values of the selected points in that part the equation.

$$E_p = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{Q} \times \frac{P}{\Delta P} = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} \quad \ldots\ldots \text{(17)}$$

However, in the case of continuous functions, calculus is used to compute the first part of equation (17), while the procedure for computing the second part of the equation is maintained or unchanged. As a result, the first component of equation (17) becomes the expression presented in equation (18).

$$\frac{\Delta Q}{\Delta P} = \frac{\delta Q}{\delta P} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(18)}$$

From equation (16) to equation (18), one observes that E_p considers the standard mathematical form of presenting the demand and supply functions (Equations (8) and (9)) in its computational procedure. Given this arrangement, what is therefore the way out for computing E_p with the casual mathematical form of expressing the demand and supply functions?

The casual mathematical form of describing the demand and supply functions suggests the dependence of price (*P*) on quantity (*Q*). Going by the requirements of equation (18), the expression for the first part of equation (17) must correspond to what appears in equation (19) since *P* is the dependent variable.

$$\frac{\delta P}{\delta Q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

However, this has always not been the practice despite the differences in the mathematical representation of the demand and supply functions of a good. In both the standard and casual mathematical forms of defining the demand and supply functions of a good, the first component of E_p is always computed using the conditions stated in equation (18). When *P* is the dependent variable, two options apply. On one hand, the functions can be rearranged to make *Q* the dependent variable before applying the conditions of equation (18). On the other hand, the functions can be maintained as expressed in terms of *P* but the inverse of equation (19) must be computed before applying the elasticity formula. With the second option, E_p is mathematically defined as in equation (20), which can be represented as in equation (21) for continuous functions.

$$E_p = \left(\frac{\Delta P}{\Delta Q}\right)^{-1} \times \frac{P}{Q} = \frac{1}{\Delta P / \Delta Q} \times \frac{P}{Q} \ldots\ldots\ldots \quad (20)$$

$$E_p = \left(\frac{\delta P}{\delta Q}\right)^{-1} \times \frac{P}{Q} = \frac{1}{\delta P / \delta Q} \times \frac{P}{Q} \ldots\ldots\ldots \quad (21)$$

From the above mathematical treatments, it is observed that in both the standard and casual mathematical forms of defining the demand and supply functions of a good, the magnitude of the first part of E_p does not change even though it is computed using different approaches. Based on this understanding, it becomes obvious that equation (18) is equal to the first part of E_p in equations (20) and (21) as detailed and indicated in equation (22).

$$\frac{\Delta Q}{\Delta P}=\frac{\delta Q}{\delta P}=\left(\frac{\Delta P}{\Delta Q}\right)^{-1}=\left(\frac{\delta P}{\delta Q}\right)^{-1}=\frac{1}{\delta P/\delta Q} \quad \ldots\ldots\ldots (22)$$

4.3 The Consumer and Producer Surpluses

This section explores the meaning of consumer and producer surpluses. It also discusses the implication of the two concepts in economic analysis, and shows how these measures are calculated based on Marshall (1890)'s reasoning. Then it offers an alternative way for computing the consumer surplus (CS) and producer surplus (PS).

CS refers to the monetary value that consumers derive from purchasing a good or service below their anticipated price or maximum willingness to pay price for it. For example, if there are 4 prospective consumers (A, B, C, and D) of good in a perfectly competitive market such that the market prevailing price is 50 for 4 units of it and it happens that A is willing to spend 200 to acquire the good, B is prepared to buy the good at 150, while C is ready to purchase it at 100, and D proposes 50 to obtain it, the surplus accruing to each of the consumers from the transaction is 150 (200 – 50 = 150) for A, 100 (150 – 50 = 100) for B, 50 (100 – 50 = 50) for C, and 0 (50 –50 = 0) for D. The horizontal summation of the surpluses accruing to the 4 consumers gives CS for the good in question as defined in equation (23). Based on equation (23), CS = 150 + 100 + 50 + 0 = **300**.

$$CS=\sum_{i=1}^{4} s_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Σ: Summation symbol
s_i: Surplus accruing to ith consumer for i = 1, 2, 3, 4

Alternatively, CS can be obtained by taking the difference between the total planned expenditure of the 4 consumers and their total actual expenditure on the good in question. This is expressed in equation (24). Based on equation (24), CS = (200 + 150 + 100 + 50) – (50 + 50 + 50 + 50) = (500) – (200) = **300**.

$$CS=\sum_{i=1}^{4} TE_i-\sum_{i=1}^{4} AE_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (24)$$

TE: Total planned expenditure of *i*th consumer for *i* = 1, 2, 3, 4
AE: Total actual expenditure of *i*th consumer for *i* = 1, 2, 3, 4

CS is equal to the difference between a consumer's maximum willingness to pay (WTP) price or the demand curve and the price that he actually pays for the good in the market. At points where the demand curve is above the market prevailing price (50), that is, to the left of the quantity offered for sale in the market (4 units), each of these purchases yields a surplus to the consumer. When the market prevailing price (50) intersects the demand curve, the surplus is zero. At points where the demand curve is below the market prevailing price (50), that is, to the right of the quantity offered for sale in the market (4 units), consumers are not willing to pay the price. Graphically, CS will correspond with the area of the triangle below the demand curve and above the market prevailing price (50). As a result, the calculation of CS for the good translates into calculating the area of a triangle, which is given in equation (25). Based on equation (25), CS = (0.5 × 4 × 150) = 2 × 150 = **300**.

$$CS = 0.5 \times (Base \times Height) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

Base = 4
Height = Maximum WTP price minus market prevailing price or actual price paid = (200 – 50) = 150

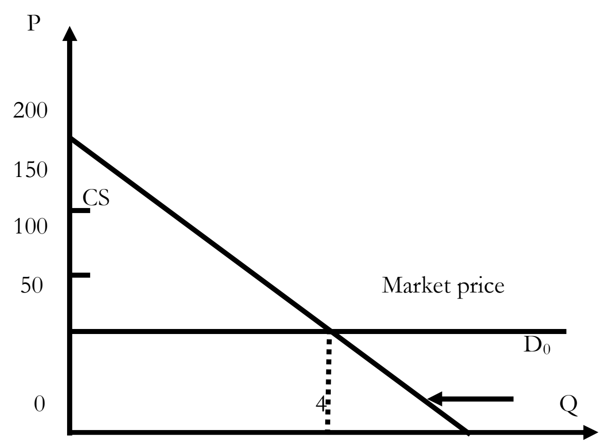

Figure 5. Consumer surplus in a perfectly competitive market

PS refers to the monetary value that producers derive from selling a good or service above their anticipated price or minimum willingness price to sell it. For instance, if there are 4 prospective producers (E, F, G, and H) of a good in a perfectly competitive market such that E intended to dispose 4 units of the good at 50 but ends up selling it at 200 in the market, F was prepared to sell the good for 100, while G was ready to offer it for sale at 150, and H insisted on 200 to sell it, the surplus accruing to each of the producers from the transaction is 200 (200 – 50 = 150) for E, 100 (200 – 100 = 100) for F, 50 (200 – 150 = 50) for G, and 0 (200 – 200 = 0) for H. The sum total of the surpluses accruing to the 4 producers gives PS for the good in question as expressed in equation (26). Based on equation (26), PS = 150 + 100 + 50 + 0 = **300**.

$$PS = \sum_{i=1}^{4} s_i \quad \text{.. (26)}$$

s_i: Surplus accruing to ith producer for i = 1, 2, 3, 4

Alternatively, PS can be obtained by taking the difference between the total actual sales of the 4 producers and their total planned sales on the good in question. This is shown in equation (27). Based on equation (27), PS = (200 + 200 + 200 + 200) – (50 + 100 + 150 + 200) = (800) – (500) = **300**.

$$PS = \sum_{i=1}^{4} AS_i - \sum_{i=1}^{4} PS_i \quad \text{.................................. (27)}$$

AS: Total actual sales of ith producer for i = 1, 2, 3, 4
PS: Total planned sales of ith producer for i = 1, 2, 3, 4

PS is equal to the difference between a producer's minimum willingness to accept (WTA) price for the sale of the good or the supply curve and the price that he actually receives for the good in the market. At points where the supply curve is below the market prevailing price (200), that is, to the left of the quantity offered for sale in the market (4 units), each of these sales procures a surplus to the producer. When the market prevailing price (200) intersects the supply curve, the surplus is zero. At points where the supply curve is above the market prevailing price (200), that is, to the right of the quantity offered for sale in the market (4 units), producers are not willing to accept the price. Geometrically, PS will coincide with the area of the triangle above the supply curve and below the market prevailing price (200). Consequently, the calculation of PS for the good corresponds with calculating the area of a triangle as indicated in equation (28). Based on equation (28), PS = (0.5 × 4 × 150) = 2 × 150 = **300**.

$$PS = 0.5 \times (Base \times Height) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

Base = 4

Height = Market prevailing price or actual price received minus minimum WTA price = (200 – 50) = 150

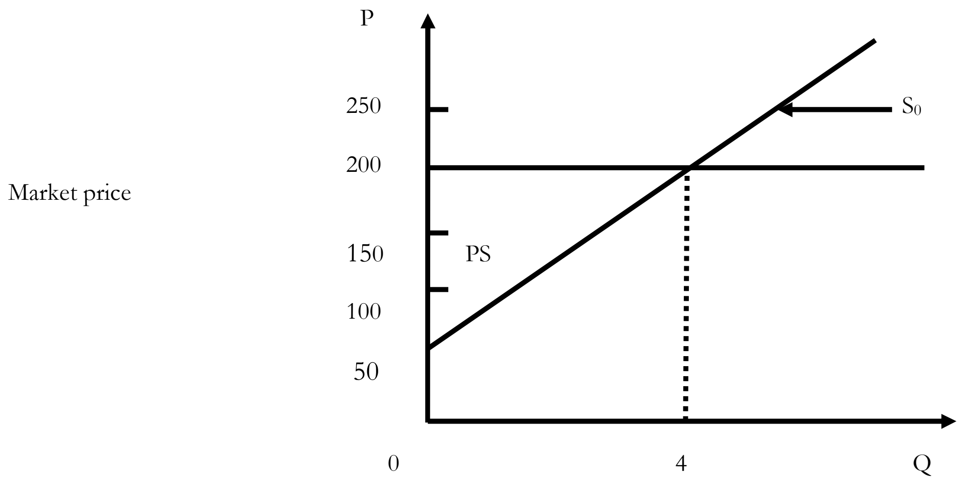

Figure 6. Producer surplus in a perfectly competitive market

The measuring of CS and PS in economic analysis is informed by the need for determining the welfare implication of the market prevailing prices *vis à vis* the maximum WTP for a good by the consumer and minimum WTA the sale of a good by the producer in the market. By interchanging the positioning of the quantity and price in the geometry of the demand and supply functions, Marshall (1890) advanced some arguments. According to Gordon (1982: 32), one of the arguments is the *wish* to interpret WTP of consumers as a measurement of utility and WTA of producers as a measurement of real cost of production. The second argument relates to the fact that producers usually think of demand in terms of quantities which will be forthcoming at different prices (Quantity-price relation) and they imagine supply in terms of the prices at which different quantities can be sold (Price-quantity relation). Though nothing was said about how consumers perceive demand and supply, this alone offered a ground for expressing the demand and supply functions either in the standard mathematical form (Quantity-price relation) or casual mathematical form (Price-quantity relation) according to Marshall (1890) as cited in Gordon (1982: 34-35). Yet, Gordon (1982: 44) contends that the main reason for Marshall (1890) to transpose the axes of the quantity demanded for or supplied and price of a good in the geometry of the demand and supply functions was the *desire* to use price as a measure of money so as to determine the implication of changes in market prices for the consumer and the producer with regard to a good in a perfectly competitive market within the framework of welfare economics, hence the specific interest in CS and PS.

Based on the underlined arguments, it is obvious that the terms *wish* and *desire* are subjective in nature, for all researchers may not have the same *wish* and *desire* in apprehending real world phenomena. The guiding principle for scientific work is positive reasoning or thinking which leans on logic, rigor, consistency, and objectivity or impartiality. Even Marshall (1890) as in Gordon (1982: 35) repeatedly emphasized the necessity of treating the demand and supply functions in consistent and commensurable terms. Besides, the way consumers perceive demand and supply is not clearly stated despite the role of consumers in the determination of the market equilibrium price and quantity, whilst producers' perception of the demand and supply is clearly underlined. Since consumers are the opposite of producers in the market and nothing was mentioned about the way they perceive demand and supply, it can be logically deduced that consumers often think of demand in terms of actual expenditures on the various quantities (Price-quantity relation) acquired and they consider supply in terms of the quantities offered for sale to them at various price levels in a competitive market (Quantity-price relation). While respecting mathematical exigencies and conforming to scientific order, the welfare magnitude of CS and PS in economic analysis can be determined and calculated as follows.

Considering the case of the 4 previous consumers, CS can be computed by taking into account the triangles bounded by the demand curves in Figure 7, where D_0 represents the maximum WTP of one of the consumers of the good in question and D_1 corresponds with the actual price of the good that all the consumers pay in the market. In this case, CS will coincide with the area of the triangle bordered by D_0 minus the area of the triangle delimited by D_1. As a result, CS is the area between D_0 and D_1 represented by triangle J. The calculation of CS takes into account the areas of 2 triangles and follows, for each, the rule for computing the area of a triangle as shown in equation (29). Applying equation (29) to the 2 triangles and taking the difference between them into consideration, CS can be obtained thus as presented in equation (30). Based on equation (30), CS = $[0.5 \times (200) \times (4)] - [0.5 \times (50) \times (4)] = 400 - 100 =$ **300**.

$$CS = 0.5 \times (Base \times Height) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(29)}$$

Base = 200 for triangle (I + J) bordered by D_0
Base = 50 for triangle (I) bordered by D_1
Height = 4 for all the triangles

$$CS = (I + J) - I = I + J - I = J \ldots\ldots\ldots\ldots\ldots\ldots \text{(30)}$$

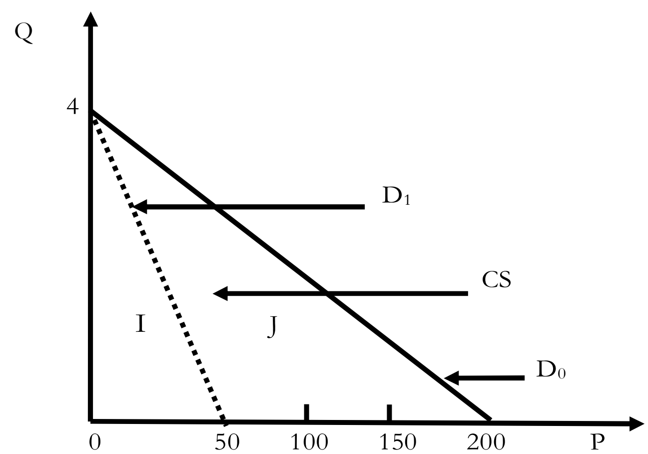

Figure 7. Consumer surplus in a perfectly competitive market

Similarly with the case of the 4 previous producers, PS can be calculated by focusing on the triangles bordered by the supply curves in Figure 8, where S_0 corresponds with the minimum WTA of one of the producers of the good in question and S_1 represents the actual price of the good that all the producers receive in the market. In this case, PS will concur with the area of the triangle delimited by S_1 minus the area of the triangle bordered by S_0. Accordingly, PS is the area between S_0 and S_1, corresponding with triangle L. The computation of PS considers the areas of 2 triangles and follows, for each, the rule for calculating the area of a triangle as shown in equation (31). Applying equation (31) to the 2 triangles and considering the difference between them, PS can be estimated thus using equation (32). Based on equation (32), PS = [0.5 × (200) × (4)] – [0.5 × (50) × (4)] = 400 – 100 = **300**.

$$CS = 0.5 \times (Base \times Height) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(31)}$$

Base = 200 for triangle (K + L) bordered by S_1
Base = 50 for triangle (K) bordered by S_0

Height = 4 for all the triangles

$$PS = (K + L) - K = K + L - K = L \text{………………..} \quad (32)$$

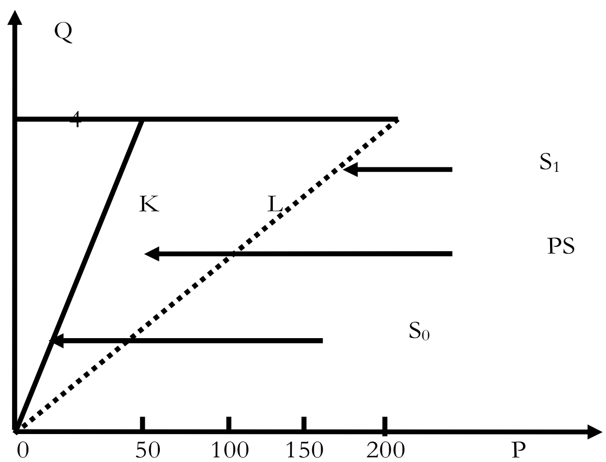

Figure 8. Producer surplus in a perfectly competitive market

4.4 Geometry of Other Analyzed Economic Phenomena

All modern microeconomic or macroeconomic analyses are guided by the interplay of the market mechanisms. After subjecting the analysis of economic phenomena to algebraic treatments, these phenomena are often visualized graphically in a 2 dimensional Cartesian plane diagram by plotting the dependent variable on the vertical axis and the independent or explanatory variable on the horizontal axis. This requirement applies to all economic analysis at both the micro and macro levels. An exploration of such economic analyses is undertaken in the following paragraphs.

In the theory of consumer behavior, the total utility (*TU*) and marginal utility (*MU*) depend on the quantity (*Q*) of the good consumed. In the graphic representation of these functions, the dependent variables (*TU* and *MU*) are plotted on the vertical axis and the independent variable (*Q*) is captured on the horizontal axis of the Cartesian plane diagram. Likewise in the theory of production, the total product (*TP*), average physical product (*APP*), and marginal physical product (*MPP*) depend on the quantity and quality of the inputs (Labor [*L*] , Capital [*K*], Land [*H*], or Entrepreneurship [*E*]) used in the production line.

When representing these functions graphically, the dependent variables (*TP, APP,* and *MPP*) always feature on the vertical axis and the independent variable (*L, K, H*, or *E*) appears on the horizontal axis of the Cartesian plane diagram.

In the analysis of revenue and cost concepts, the total revenue (*TR*), marginal revenue (*MR*) total cost (*TC*), average total cost (*ATC*), total variable cost (*TVC*), total fixed cost (*TFC*) average fixed cost (*AFC*), and marginal cost (*MC*) depend on the quantity (*Q*) of the good sold in the market. The graphic representation of these functions requires that the dependent variables (*TR, MR, TC, ATC, TVC, TFC, AFC* and *MC*) appears on the vertical axis and the independent variable (*Q*) features on the horizontal axis of the Cartesian plane diagram. And finally in the Keynesian analysis of national income (*NY*) or gross domestic product (*GDP*) equilibrium determination of an economy where the level of consumption (*C*), level of investment (*I*), level of government spending (*G*), and level of net exports (*X–M*) depend on *GDP* of that economy, these dependent variables are always captured on the vertical axis and the explanatory variable (*GDP*) is plotted on the horizontal axis when representing the phenomenon graphically on a Cartesian plane diagram.

5. Conclusion and Recommendations

Considering mathematical logic and rigor, this paper proves that although the demand and supply functions are expressed in two optional forms where either *Q* (Quantity demanded for or supplied of) or *P* (Own-price of the good) is the dependent variable, the standard algebraic form is one in which *Q* is the dependent variable. Yet despite the adoption of this structural exposition, the graphic representation of these two economic phenomena often considers the case in which *P* is the dependent variable of the model being described. The imposition of a graphic representation of the demand and supply functions where the positioning of the dependent and explanatory variables is interchanged for the standard algebraic form in which *Q* depends on *P* is a serious violation of mathematical rigor and consistency. This lures some elements of bias and mathematical fraud in the analysis of economic phenomena. Even when the graphic representation of the demand and supply functions is compared with functions of other economic models, the observed inconsistency persists. All other economic models have their dependent variable graphically captured on the vertical axis and their explanatory variable featuring on the horizontal axis, except the demand and supply functions, where all the time *P* appears on the vertical axis and *Q* features on the horizontal axis of the Cartesian plane diagram irrespective of the algebraic form chosen to model the phenomena. In view of the aforementioned, the following suggestions are advanced.

1. If both the standard and casual forms of describing the demand and supply functions are to be maintained in economic analyses, then the geometry of each form should conform to the structural arrangement of its variables. This implies that for the standard form of defining the demand and supply functions, the dependent variable *Q* should always appear on the vertical axis and the explanatory variable *P* on the horizontal axis of the Cartesian plane diagram. The same rule should apply for the casual form of describing the demand and supply functions by plotting the dependent variable *P* on the vertical axis and the explanatory variable *Q* on the horizontal axis of the Cartesian plane diagram so as to satisfy the mathematical requirements of logic, rigor and consistency regarding the definition of a function and its corresponding graphic representation.

2. Although the laws of demand and supply are verified with the two forms of expressing the demand and supply functions, the standard form where *Q* depends on *P* is most appealing and appropriate in describing the phenomena. Therefore the plotting of *Q* on the vertical axis and *P* on the horizontal axis of the Cartesian plane diagram should be most appropriate based on mathematical requirements of logic, rigor and consistency that insist on placing the dependent variable of a function on the vertical axis and its explanatory variable on the horizontal axis in a 2 dimensional graphic representation diagram. Furthermore, all other microeconomic or macroeconomic models describing a functional relationship between a dependent variable and an explanatory variable of a given phenomenon have their dependent variable always plotted on the vertical axis and their explanatory variable on the horizontal axis when represented graphically on a Cartesian plane diagram. The demand and supply being the bedrock of all economic analyses, the graphic representation of their functions should not be on any grounds the exception that confirms the rule.

3. The concepts of price elasticity of demand and supply measure the responsiveness of quantity demanded for or supplied of a good to slight changes in the price of the good under consideration. In both the standard and casual algebraic forms of describing the demand and supply functions of a good o service, the price-elasticity is computed by focusing on the effect of price changes on the quantity demanded for or supplied of the good. This

presupposes the dependence of *Q* on *P* as the standard form of defining the demand and supply functions suggests.

4. By obeying mathematical requirements rather than leaning on personal judgment or affinities, the welfare implications of consumer surplus on the consumer and producer surplus on the producer can still be assessed with the standard (Quantity-price relation) form of describing the demand and supply functions.

References

Cournot, AA (1838): *Recherches sur les principes mathématiques de la theorie des richesses*, Paris: Hachette, translated by NT Bacon (1897): *Researches into the mathematical principles of the theory of Wealth,* New York: Kelley, 1960.

Davenant, C (1699): *An essay upon the probable methods of making a people gainers in the balance of trade*. Reprinted in C Whitworth [Ed] (1771): *Political and commercial works of Charles Davenant*, London: Macmillan and Co. Ltd.

Dupuit, AJE (1844): De la mesure de l'utilité des travaux publics, *Annales des Ponts et Chaussées*, Vol. 8:332-375, translated by RH Barback (1952): On the measurement of the utility of public works, *International Economic Papers*, 2: 83-110.

Endres, AM (1987): The King-Davenant 'law' in classical economics, *History of Political Economy*, Vol. 19 (4): 621-638. Retrieved on 26 September 2017 from https://www.researchgate.net/.../AnthonyEndres2/publication/..._The_KingDavenant_'Law'_in_Classical_Economics/links/.../The-King-Davenant-Law-in-Classical-Economics.pdf?

Gordon, S (1982): Why did Marshall transpose the axes?, *Eastern Economic Journal*, Vol. 8(1): 31-45. Retrieved on 22 June 2017 from econpapers.repec.org/article/eejeeconj/v_3…..i_3a1_3ap_3a31-45.htm

Humphrey, TM (1992): Marshallian cross diagrams and their uses before Alfred Marshall, *Federal Reserve Bank of Richmond Economic Review*, Vol. 78(2): 3-23. Retrieved on 14 October 2017 from: https://fraser.stlouisfed.org /files/docs/publications/frbrichreview/rev_frbrich199203.pdf

ibn Taimiyah, AA (1381): *Majmu' fatawa Shaikh al Islam Ahmad b. Taimiyah*, Vol.8, Riyadh: al Riyadh Press.

Islahi, AA (1983): Ibn Taimiyah's concept of market mechanism, *Journal of Research in Islamic Economics,* Vol. 2 (2):51-60. Retrieved on 26 September 2017 from https://mpra.ub.uni-muenchen.de/18466/1/Concept_of_Market_

Mechanism.pdf
Jenkin, HCF (1870): The graphic representation of the laws of supply and demand, and their application to labour, *Recess Studies*, Ch. 6: 151–185.
King, G. (1696): Natural and political observations and conclusions upon the state and condition of England. In GE Barnett [Ed] (1936): *Two Tracts by Gregory King*, Baltimore: Johns Hopkings Press.
Locke, J (1691): Some considerations on the consequences of the lowering of interest and the raising of the value of money. In a letter sent to a member of parliament, Vol. 4 (B). In *The works of John Locke*, 9 Volumes, 12e, Vol. 4, London: Longman and Co, 1824. Retrieved on 14 October 2017 from: https://books.google.fr/books?id=_EM1AA....Locke(1691)…+the+value+of+money....
Marshall, A (1890): *Principles of economics*, Vol.1, London: Macmillan and Co. Retrieved on 14 October 2017 from: https://ia601600.us.archive.org/15/items/in.....2015.217841/2015.217841.Principles-Of.pdf
Mill, JS (1767): *Principles of political economy*, New York: Augustus M Kelley. Retrieved on 14 October 2017 from: https://eet.pixel-online.org/files/...../Mill....Principles.......20Political%20Economy.pdf
Rau, KH (1841): *Grundsätze der Volkswirthschaftslehre*, 4e, Heidelberg: CF Winter.
Ricardo, D (1817): *On the principles of political economy and taxation*, 3e, 1821, Kitchener, Ontario: Batoche Books. Retrieved on 26 September 2017 from www.socserv.mcmaster.ca/...ugcm/.....ricardo/Principles.pdf
Smith, A (1776): *An inquiry into the nature and causes of the wealth of nations*, New York: Penguin Books
von Mangoldt, HKE (1863): *Grundriss der Volkswirthschaftslehre*, Stuttgart: Engelhorn, translated by E Henderson (1962): The exchange ratio of goods, in *International Economic Papers*, 11: 32-59, London: Macmillan and Co. Ltd.