An Examination of the Occurrence of Speculative Bubbles in the US Stock Markets

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Abstract
The paper will investigate the possibility of the formation of a speculative bubble in the U.S. stock markets after the 2007-2008 crises. The work is initiated by the observation of historically high equity prices in the stock markets as pointed out by Professor Robert J. Shiller in his new edition of his book 'Irrational Exuberance'. Until recently the development of a methodology which can be used to detect bubble in real time has been challenging and previous work has mainly focused on ex-post econometric tests in time series that contain only one bubble. The paper uses the recent methodology developed by Philips et. al. that fixes for these deficiencies and enable real time monitoring of statistically significant exuberance and multiple collapsing bubbles within a single time series. Both NASDAQ and S&P 500 indexes are considered and investigation regarding exuberance that cannot be explained by the market fundamentals is conducted.

Keywords: Differential Equations, Price exuberance, US stock markets, Real time monitoring

JEL Classification: C58, G15

1. Introduction

The idea that that price movements are possibly instigated by self-fulfilling prophecies of market participants has inherently intrigued economic thinkers of free markets-a phenomena called 'bubble' to indicate its origination from events that are rather extraneous to the market [7]. Empirical observations like the ones of the South Sea bubble in UK and most distinguishably the Dutch tulip bulb episode (see [9] for a thorough analysis) have seemingly challenged the fundamental theoretical framework when it comes to modelling price path evolutions in equity markets.

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Perhaps the most challenging problem when it comes to building empirical models that test for Speculative Bubbles has, until recently, remained the formulation of tests that can monitor the bubble formation in real time contrary to classical ex-post investigation. Economic historians have lately observed that financial crises are usually led by an asset market bubble or unusual growth of liquidity levels [8] [1] [14]. It is therefore of crucial importance for policy makers to be able to constantly monitor the equity markets for possible inflationary behaviour beyond the fundamental predicted values. For instance, Jean Claude Trichet who served as former president of ECB has considered this '…to be one of the most challenging issues facing a modern central bank at the beginning of the 21st century'; a task agreed upon until recently as impossible by the majority of economists due to the lack of the suitable econometric apparatus [10]. This paper will attempt to overcome these shortcomings by employing a recent methodology developed by PCB Philips et. al. ([14], [15]) which enables the detection of bubbles in their inflationary stage. The motivation for this analysis is originated by recent developments in equity prices; in his recent book 'Irrational Exuberance', Robert J. Shiller [18] maintains that there is possibly a speculative bubble forming in the U.S. stock markets in the aftermath of the 2007-2008 crises. These arguments seem to be indeed supported by new records with regards to the magnitude of nominal stock prices. Moreover, surveys conducted regularly by Yale University regarding confidence in the markets seem to further reinforce a reintroduced over optimism in the stock markets [18].

2. Theoritical Model and Literature

Perhaps the best way to give the definition of a 'speculative bubble' is to present the fundamental theoretical model; this should also serve complimentary to attain a better understanding with regards to the evolutionary process of developing the econometric tests that are presented in the subsequent parts of this chapter. The most widely applied model that allows for rational bubbles, is the present value model which interprets stock price movements as a reflection of information about expected dividends payouts. The model utilized follows the one used by Blanchard and Watson [3] or Campbell, Lo and MacKinlay [5] and its adaption by Homm and Breitung [11]. Assuming the condition of no arbitrage the following relation is obtained regarding the fundamental price of an asset.

$$P_t = \frac{E_t[P_{t+1} + D_{t+1}]}{1 + R} \qquad (2.1)$$

where P_t is the stock price at period *t*, D_{t+1} denotes the dividend at period *t*, *R* is the risk-free rate, and $E_t[.]$ denotes the expectation conditional on the information at time *t*. Using forward iteration to solve equation (2.1), the following relation about the fundamental price is derived:

$$P_t^f = \sum_{i=1}^{\infty} \frac{1}{(1+R)^i} E_t[D_{t+i}] \tag{2.2}$$

Equation (2.2) defines the fundamental price of the asset as the present value of all expected dividend payments. If we impose the transversality condition the following is obtained:

$$\lim_{k\to\infty} E_t\left[\frac{1}{(1+R)^k} P_{t+k}\right] = 0 \tag{2.3}$$

which ensures that $P_t = P_t^f$ is a unique solution for (2.1); it rules out the possibility of a bubble conditional on (2.3) holding. If (2.3) does not hold, P_t^f is not a unique solution to (2.1). Let $\{B_t\}_{1=1}^{\infty}$ be a process that satisfies the property

$$E_t[B_{t+1}] = (1+R)B_t \tag{2.4}$$

By adding B_t to P_t^f will give infinite solutions to (2.1) of the form

$$P_t = P_t^f + B_t \tag{2.5}$$

where $\{B_t\}_{1=1}^{\infty}$satisfies equation (2.4). The last equation gives the price as a decomposition of the fundamental component,P_t^f and B_t which is referred to as the bubble component. Alternatively, by solving (2.1) forwards the subsequent equation is attained

$$P_t = E_t\left[\sum_{i=1}^{T-i} \frac{1}{(1+R)^i} D_{t+i}\right] + E_t\left[\frac{1}{(1+R)^{T-i}} P_t\right] \tag{2.6}$$

where $E_t\left[\sum_{i=1}^{T-i} \frac{1}{(1+R)^i} D_{t+i}\right] = P_t^f$ and $E_t\left[\frac{1}{(1+R)^{T-i}} P_t\right] = B_t$reflecting the expected capital gains through anticipated price changes. A rational investor would engage in buying an overpriced stock if she thinks that she will be compensated through price increases for the additional payment ofB_t.

The statistical characteristics of P_t are dependent on those of P_t^f andB_t. For instance if D_t is an I(1) process, P_t^f is also I(1). The bubble component B_t carries the explosive property and brings changes in the price P_t attributed to bubble movements over the fundamental componentP_t. In the case of an abstence of a bubble, $B_t = 0$. Therefore, the price of the commodity is determined only by market fundamentals and expression (2.5) becomes $P_t = P_t^f$. On the other hand, if P_t^f is I(1), current prices are also I(1). If $B_t \neq 0$, current prices will exhibit the explosive behavior given in (2.4).

2.1. Literature

The majority of literature on detecting bubbles is concentrated in applying Integration/Cointegration tests. Diba and Grossman [10] examined dividends and stocks for nonstationarity to reject the hypothesis of a bubble existing in the series. The method used to detect explosive behavior in time series is originated from the following Augmented Dickey-Fuller (ADF) regression equation:

$$\Delta y_t = \alpha_{r1,r2} + \beta_{r1,r2} y_{t-1} + \sum_{i=1}^{k} \psi_{r1,r2}^{i} \Delta y_{t-1} + \varepsilon_t \qquad (2.5)$$

where y_t denotes the logged time series which in this case would be the price of the commodity studied at time t, $\varepsilon_t \overset{iid}{\sim} N(0, \sigma_{r_1,r_2}^2)$ while r_1 and r_2 denote the starting point and ending point of a subsample period within the total sample. Their findings indicate that there is no bubble in the S&P500 time series. Evans [6] criticized this approach by showing that the unit root and cointegration tests are not efficient in distinguishing between a stationary process and a process that exhibits periodically collapsing behavior. Even though Evans showed that unit roots tests lack the power to test for bubbles, he also was not able to proof the existence of bubbles in the S&P500 series. Following Evan's criticism, numerous research papers have attempted to overcome the challenge of detecting periodically collapsing bubbles. Gurkayak [10] gives a thorough review of competing models. More recent methodologies developed intending to overcome the challenge of collapsing bubbles is reviewed thoroughly by Homm and Breitung [11]. Philips et. al [13] introduced sequential unit root tests intending to increase the testing power. Homm and Breitung [11] showed that the Philips et al [13] tests outperformed in terms of power other similar tests but failed to deal with the issue of more than one bubble in the markets. The recently developed tests by Philips et al [14] fixed for those shortcomings and will be the ones adopted in this paper.

3. Methodology

A recent test introduced in Phillips et al [14] [15] will be utilized. The test is a generalized version of the SADF test (GSADF) and enables the examination of multiple bubbles within the same sample. The suitability and relative superiority of the GSADF test in detecting multiple price exuberance within the same series can be rationalized by the periodic booms and boosts which characterize price volatilities in the stock market. For convenience, equation 2.5 is produced again in order to explain the evolution of methodology used.

$$\Delta y_t = \alpha_{r1,r2} + \beta_{r1,r2} y_{t-1} + \sum_{i=1}^{k} \psi_{r1,r2}^{i} \Delta y_{t-1} + \varepsilon_t \qquad (3.1)$$

3.1. The SADF method

Studying the SADF test is of crucial importance when it comes to understanding the GSADF method and will, therefore, be explained thoroughly in this dissertation. In order to account for the effect of a price collapse on the performance of the test, Phillips et al (2011) applied a recursive procedure which consists in repeated estimations of the ADF test in subsamples of the data. Specifically, (3.1) is estimated by holding r_1 constant at 0 (the beginning of the sample) while $r_2$increases continuously from r_0, which is the minimum window size, to one, which indicates the last observation of the series. This forward expanding sample procedure produces multiple regressions; the first regression utilizes the subsample from the first observation to the kth observation (determined by the value of r_0 where $\tau_0 = [Tr_0]$) followed by the second regression that includes the $(k+1)^{th}$ observation. Equation (3.1) would transform to the series of equations:

$$\Delta y_0 = \alpha_{rc,r0} + \beta_{rc,r0} y_{t-1} + \sum_{i=1}^{k} \psi_{rc,r0}^{i} \Delta y_{t-1} + \varepsilon_t$$

$$\ldots \quad \Delta y_T = \alpha_{rc,rT} + \beta_{rc,rT} y_{t-1} + \sum_{i=1}^{k} \psi_{rc,rT}^{i} \Delta y_{t-1} + \varepsilon_t \qquad (3.2)$$

(3.2) where r_c is fixed to the first observation and $r_T = 1$ and includes the last observation of the series. Each equation generates an ADF test statistics$\{ADF_0^{r0}, ADF_0^{r1} \ldots ADF_0^{rT}\}$, where $ADF_0^{r0}$is the test statistics of the initial subsample generated by the window r_0 while ADF_0^{rT} is the test statistic of the whole sample. Phillips, Shi and Yu [13] utilises the sample of ADF statistics to generate the following test statistic for all the regressions called sup ADF *(SADF)*:

$$SADF_{(r_0)} = \sup_{r_2 \epsilon [r_0,1]} ADF_0^{r_2} \qquad (3.3)$$

The distribution of the SADF statistic is given by:

$$\sup_{r_2 \epsilon [r_0,1]} \frac{\int_0^{r_2} W dW}{\int_0^{r_2} W^2} \qquad (3.4)$$

The test found broad empirical applications and performs well when it comes to investigating single bubble episodes. Through simulations of several price scenarios Homm and Breitung [11] show that SADF performs better than alternative approaches such as Kim [12], Bhargava [2] and Busseti and Teylor [4]. However, it seems to perform poorly and inconsistently when it comes to testing for more than one boom-bust episode Phillips et al [14].

3.2. The Generalized *SADF (GSADF)* test

The test that will be used in this paper is the recent one by Phillips et al [14] [15] which is the generalized version of the *SADF* test *(GSADF)* and allows for the test of multiple bubbles within the same sample. The test covers a significantly larger number of subsamples by allowing both $r1$ and $r2$ to change and therefore generates substantially more power than the *SADF* test. Following the logic for deriving the *SADF* statistic, the *GSADF* statistic is derived as:

$$SADF_{(r_0)} = \sup_{r_2 \epsilon [r_0,1]} ADF_0^{r_2} \quad (3.5)$$

The limit distribution, under the null hypothesis, of the GSADF statistic is:

$$\sup_{r_2 \epsilon [r_0,1], r_1 \epsilon [0, r_2 - r_0]} \left\{ \frac{\frac{1}{2} r_w \left[W(r_2)^2 - W(r_1)^2 - r_w \right] - \int_{r_1}^{r_2} W(r) d_r \left[W(r_2) - W(r_1) \right]}{r_w^{1/2} \left\{ r_w \int_{r_1}^{r_2} W(r)^2 d_r - \left[\int_{r_1}^{r_2} W(r)\, d_r \right]^2 \right\}^{1/2}} \right\} \quad (3.6)$$

3.3. Details on implementing the methodology

The majority of the studies applying the Phillips et al [14] [15] procedure use the asymptotic *SADF* and *GSADF* critical values that are provided by the original paper. However, in my investigation I have simulated critical values for the specific sample length intending to obtain more accurate conclusions; in particular it is intended to obtain accurate dates for the beginning and collapse of bubbles. Firstly, the computation of *BSADF* and *GSADF* test statistic requires the determination minimum window size r_0 and the autoregressive lag length k. The size of the minimum window should be large enough so it can provide initial estimation for the subsample but not too large so that short exuberance episodes are missed. As in Phillips et al [14] the minimum window size equals 36 observations apart from the tests conducted on historical S&P500 time series Price/Dividend ratio for which an initial window size of 81 observations was implemented. Regarding lag length k it is found by performing a unit root test for each series. For all the series a lag length of 0 was chosen. Finally, because the limit distributions of the *SADF*, *BSADF* and *GSADF* statistics are nonstandard and dependable on the minimum window size, critical values were obtained through Monte Carlo simulations as aforementioned. Finite sample of critical values were obtained by generating 2,000 random walk processes with N(0,1) errors. All the simulations were attained in MATLAB.

4. Data

This study is primarily focalized in investigating stock price behavior in U.S. markets. With regards to U.S. *NASDAQ* and *S&P500* composite indices are analyzed. Composite tests are performed using both weekly and daily frequencies while for S&P500 apart from daily, monthly frequencies are used as well to draw conclusions about historical stock prices that date since 1881. Daily and weekly Price Indices for NASDAQ as well as daily S&P500 are obtained from Datastream International. S&P500 monthly data is obtained from Robert Shiller's website and it is the same used in his three editions of Irrational Exuberance (Shiller 2000, 2005, 2015). Daily and weekly Dividend Yields are taken from Datastream International. Dividends composite series are computed from the series of Dividend Yields and Price Indices. Monthly Consumer Price Index (CPI) series for U.S. are collected from the Federal Reserve Bank of St. Louis in order to convert nominal series to real series. The sample for monthly S&P500 composite covers the period from January 1881 to June 2015 and consists of 1734 observations. Weekly series of NASDAQ composite cover the period from 26 February 1982 to 4 September 2015 and consist of 1750 observations. Finally 1481 observations were collected for NASDAQ daily which cover the span of time from January the 1st 2010 to September the 4th 2015.

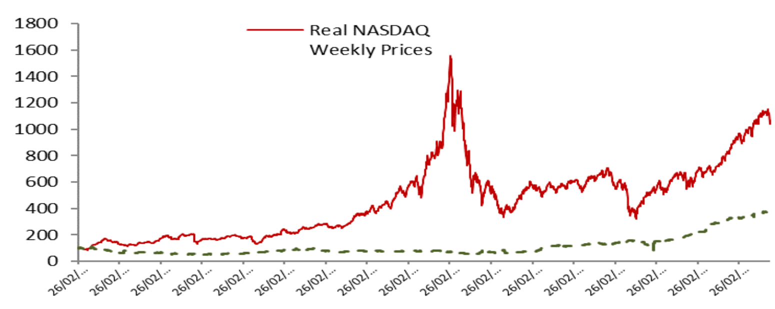

Figure 4.1 Real Weekly NASDAQ prices and dividends normalized to 100

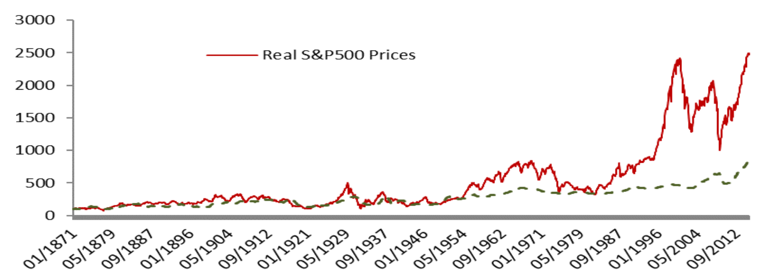

Figure 4.2 Real Monthly S&P500 prices and dividends normalized to 100

5. Empirical Results

Most of empirical research on the domain is conducted with quarterly or monthly frequency. As showed by [11], low frequency data can hide periods of explosive behaviour. Apart from attempting to answer the question of whether there are speculative bubbles emerging in the U.S. stock market, it is of equally crucial importance to determine to what extend the frequency of the data used can affect those results. Following the description given in the previous chapter, the results that will be displayed in this dissertation cover daily data from 2010 while weekly data covers periods from 1982 and 1994 for NASDAQ. Additional tests were conducted with weekly and monthly frequencies for the periods from 2010 to 2015 as a comparison to the conclusions from daily frequencies. GSADF sequential test statistics obtained from weekly data for the 2010-2015 samples were very similar to those that can be observed from the subsample of 2010-2015 of the whole weekly series. This is not surprising since the GSADF procedure allows for a rolling window that is not fixed to the beginning of the sample. However, when compared to results from daily frequencies the power of the test decreases as it will be demonstrated below.

For each series the sequential values of the GASDF statistic were obtained through Monte Carlo simulations as well as the corresponding sequences of the 95% significant level sequential critical values. This exercise is particularly useful for investigating more thoroughly the properties of possible bubbles and their evolution by giving their beginning and collapse dates as opposed to just determining their existence or not in a given series.

5.1. U.S. Stock market

Table 5.1 gives a summary of the data with regards to testing both S&P500 and NASDAQ composite series. As it can be observed, for both S&P500 Monthly and

NASDAQ Weekly the null hypothesis of no bubble is rejected at the 99% critical level. On the other hand, the null hypothesis cannot be rejected for S&P500 as well as NASDAQ daily series.

Table 5.1 GSADF test results for S&P500 and NASDAQ

Series	Max GSADF Statistic	95% Critical Value	99% Critical Value	Observations
S&P500 Monthly	3.44			1734
S&P500 Daily	1.42			1481
NASDAQ Weekly	3.46			1750
NASDAQ Daily	0.925			1481

A more thorough analysis can be obtained by observing the following figures which aim to Date Stamp the start and end date of the bubbles. Figure 1 gives the monthly historical Price Dividend ratio for S&P500. Well known periods of abnormal price exuberance can be identified among others; *the great crash* (1928-1929), *the postwar boom* (1954), *black Monday* (1986-1987), and the dot-com bubble (1995-2001); similar excessive price volatilities are present in NASDAQ Weekly-most notably the dot-com bubble.

It should be noted that determining whether a series is exhibiting explosive price behavior by comparing its test statistic to the sample critical value is less powerful than compering the backward BSADF sequence to the sequential critical values. For instance, the maximum value of GSADF statistic for S&P500 daily seems to be smaller than 95% critical value; however, 'short lived' bubbles can be detected by applying the BSDAF procedure in the in the period of May 2010.

Examining the evolution of BSADF sequences there seems to be little to no evidence of bubbles in both series over the last five years apart from isolated cases of short period bubbles. It is of significant importance for these episodes not to be misinterpreted and be generalized to lead to conclusions about the entire market, especially when high frequency data is used. Such episodes failed to show persistence over more than 2-3 days which accounts for only 0.1% of the number of observations considered. The significantly high nominal prices seem to have been followed closely by a steady increase in the price of dividend payments that enables them to still be explained within the framework of the market fundamentals.

Despite the inconclusive results regarding the presence of bubbles during the last 5 years in U.S. stock markets, a few interesting observations are worth considering. Both dividend price ratio series seem to be characterized by a rather constant increase throughout the last couple of years. The inability to suitably conclude with regards to statistical assessments that can confirm the presence of bubbles does not underestimate continual monitoring of the markets to verify for the persistence of such patterns.

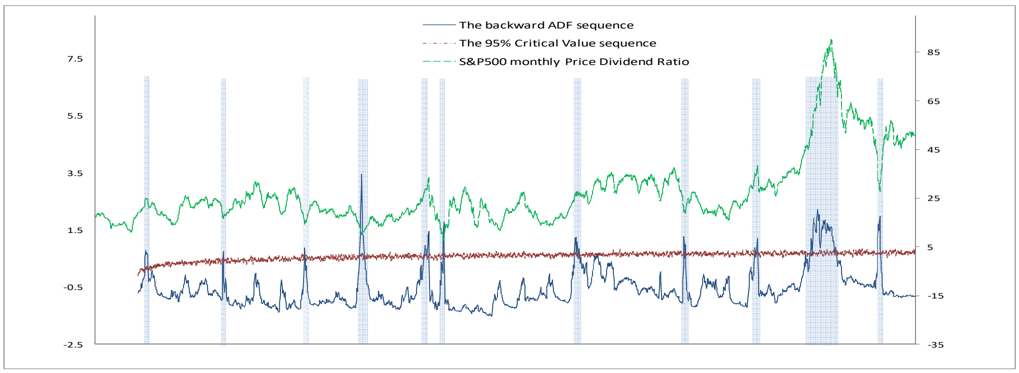

Figure 5.1 Date stamping periods of exuberance in monthly S&P500 time series

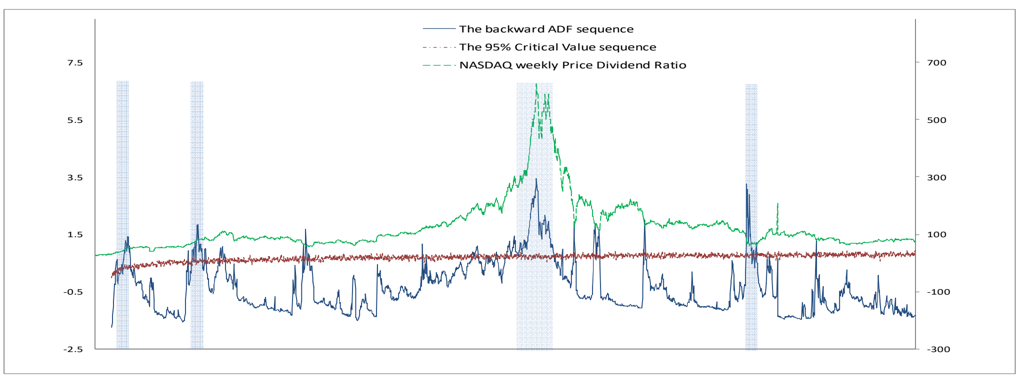

Figure 5.2 Date stamping periods of exuberance in the weekly NASDAQ time series

6. Conclusions

In this paper the main aim was to implement reliable methodologies intending to conclude on two important questions that were introduced. The hypothesis that there exists a speculative in U.S. markets was tackled by investigating on composite price and dividend indices of NASDAQ and S&P500. For S&P500 historical monthly data apart from a more recent one of daily frequency was employed. Historical instances of price explosiveness were successfully identified. Similarly for NASDAQ weekly and daily time series were implemented. For both the daily series there was no significant prove that can support the notion that a speculative bubble exists in the stock markets. However, it was observed constant increase in the price dividend ratio over the last couple of years-not significant enough that could be statistically assessed-nevertheless deserving of careful future assessment.

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