Testing for Heteroskedasticity on the Bucharest Stock Exchange

Phd. Radu Lupu
ASE-REI Bucureşti
Cercetator Iulia Lupu
Centrul Victor Slavescu

The ARCH type of models is a notorious family of models proven to be suitable for predicting financial returns. Their notoriety flourished after Bollerslev (1986) developed the econometric Generalized ARCH model (GARCH). This paper provides a presentation of the main characteristics of the modeling of financial returns with the objective to calibrate an EGARCH (Exponential GARCH) model for the logarithmic returns of the Romanian composite index BET-C on the stocks listed at the Bucharest Stock Exchange. We continue a previous study Lupu (2005) to model the statistical properties of these returns in comparison with the main non-normality properties found in previous research for the US stock index. We found that these properties are generally held on the Romanian market and this provides us reasons to trust the opportunity of an EGARCH model. The article provides the testing of the predictive power of this model for the Romanian index by calibrating the model and then evaluate its performance on an out of sample test.

Keywords: Exponential GARCH, financial econometrics, Romanian stock exchange
JEL codes: C13, C32, C51, C52

1. GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY

GARCH (*Generalized AutoRegressive Conditional Heteroskedasticity*) is a family of models that allow to include the most important properties of the returns. The problem of these models is that the parameters' estimation needs nonlinear statistic models with a high degree of complexity.

The simplest dynamic form of variance in GARCH form is:

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \beta \sigma_{t+1}^2 \sigma_t^2 \text{, where } \alpha + \beta < 1. \qquad (1)$$

To measure the risk is necessary to check if we need an equation of dynamic variance which will take into account the existence of a stable unconditioned distribution. Obviously, passing-by the long run mean become an increasing problem for long run forecasting and less important for daily forecasting.

If we intend to forecast the return variance after k days on the bases of the information obtained at the end of current transaction session, we will observe that the mean of these values would be:

$$E_t\left[\sigma^2_{t+k}\right]=\sigma^2+\alpha E_t\left[R^2_{t+k-1}-\sigma^2\right]+\beta E_t\left[\sigma^2_{t+k-1}-\sigma^2\right]$$
$$E_t\left[\sigma^2_{t+k}\right]-\sigma^2=\alpha E_t\left[\sigma^2_{t+k-1}z^2_{t+k-1}-\sigma^2\right]+\beta E_t\left[\sigma^2_{t+k-1}-\sigma^2\right]=(\alpha+\beta)\left(E_t\left[\sigma^2_{t+k-1}\right]-\sigma^2\right)$$
$$E_t\left[\sigma^2_{t+k}\right]-\sigma^2=(\alpha+\beta)^{k-1}\left(E_t\left[\sigma^2_{t+1}\right]-\sigma^2\right)=(\alpha+\beta)^{k-1}\left(\sigma^2_{t+1}-\sigma^2\right)$$
(2)

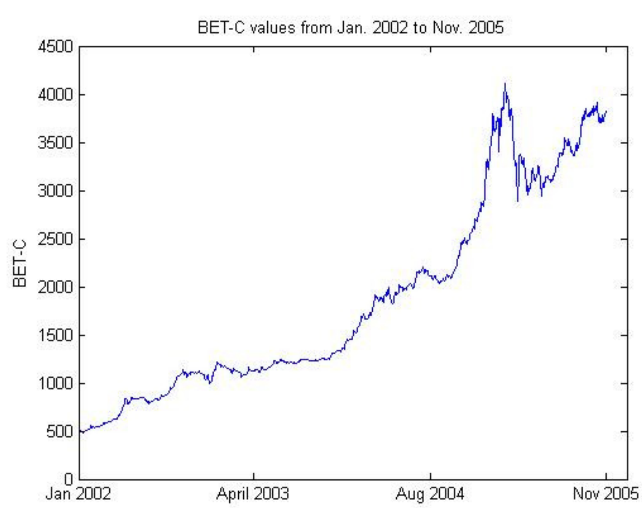
We considered that $E_t[x]$ is the mean (the expectation regarding the value) of x, computed on the basis of the information that we have at the moment t. We assumed that $R_{t+1}=\sigma_{t+1}z_{t+1}$ and than we solved up until σ^2_{t+1}.

$\alpha+\beta$ is known as the persistence of the model. A high persistence (when $\alpha+\beta$ is closed to 1) makes the shocks that push the variance away from the long term mean to have an important effect on future variances although eventually the forecasted long term variance will be the average variance (for the unconditional variance). We notice that $E_t\left[\sigma^2_{t+k}\right]-\sigma^2$ (which stands for the distance from the probable value of the variance k periods in the future) is the smaller as $\alpha+\beta$ is smaller than 1. The size of this deviation from the long term mean is given by the expected deviation at t+1 multiplied by the persistence. If we record a high variance in the present then the value from tomorrow will also be high while the value k periods from now will also be high as long as we have a high persistence, ($\alpha+\beta$ is closed to 1). When $\alpha+\beta$ = 1 the best estimate of the variance from t+k is the variance at t+1.

2. **PROPERTIES OF RETURNS**

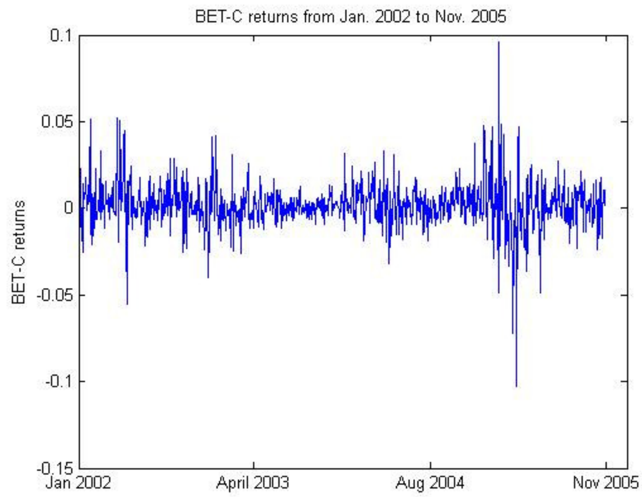
We will assume that return series are stationary processes. This Figure 1 illustrates an equity price series. In this case, it shows daily closing values of the BET-C index from 3rd of January 2002 until 17th of November 2005. Notice that there appears to be no long-run average level about which the series evolves. This is evidence of a non-stationary time series.

Figure 1 – BET-C values from January 2002 to November 2005



The following figure, however, illustrates the continuously compounded returns associated with the same price series. In contrast, the returns appear to be quite stable over time, and the transformation from prices to returns has produced a stationary time series.

Figure 2 – BET-C returns from January 2002 to November 2005



3. THE MODEL

3.1 EGARCH(P,Q) Conditional Variance

The Matlab package that has been used for this analysis uses the EGARCH(P,Q) model:

$$\log\left(\sigma_t^2\right) = k + \sum_{i=1}^{P} G_i \log\left(\sigma_{t-i}^2\right) + \sum_{j=1}^{Q} A_j \left[\frac{\left|\varepsilon_{t-j}\right|}{\sigma_{t-j}} - E\left\{\frac{\left|\varepsilon_{t-j}\right|}{\sigma_{t-j}}\right\}\right] + \sum_{j=1}^{Q} L_j \left(\frac{\varepsilon_{t-j}}{\sigma_{t-j}}\right)$$

(3)

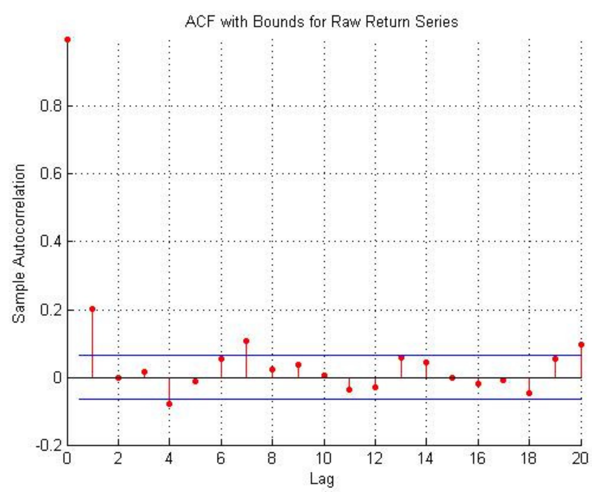
EGARCH(P,Q) models are treated as ARMA(P,Q) models for $\log\left(\sigma_t^2\right)$. We observe that EGARCH models are fundamentally different from the classical GARCH model in that the standardized innovation, z_t, serves as the forcing variable for both the conditional variance and the error. On the other hand the GARCH model allows for volatility clustering (i.e., persistence) by a combination of the G_i and A_j terms, whereas persistence in EGARCH models is entirely captured by the G_i terms.

The following analysis is checking for the basic ARCH type of models setting. We will check for the autocorrelation of returns and the autocorrelation of squared returns as a proxy for variance.

3.2 Checking for correlation in the return series

The Figure 3 shows the sample autocorrelation function of the returns, along with the upper and lower standard deviation confidence bounds, based on the assumption that all autocorrelations are zero beyond lag zero. We used lags until the 20th lag and we can notice that the returns exhibit significant correlation at the first, the fourth and the seventh lag. We see that the first lag is significantly out of the 95% confidence interval which is why we will use an E-Garch model with one lag for the mean equation.

Figure 3 – Autocorrelation Function for Raw Return Series



3.3 Checking for correlation in the squared returns

The ACF of the squared returns may still indicate significant correlation and persistence in the second-order moments.

Figure 4 – Autocorrelation Function of the Squared Returns

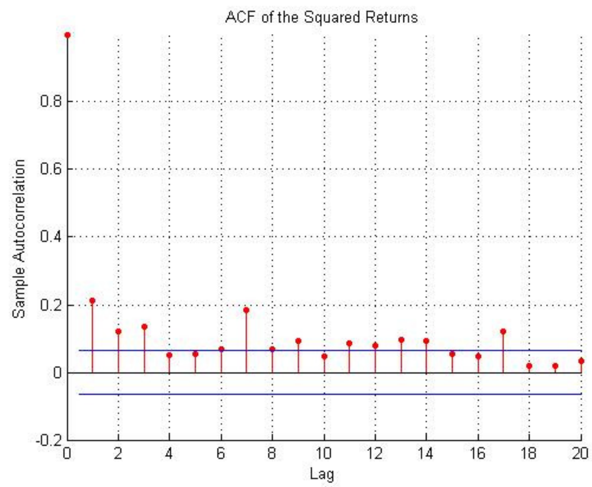


Figure 4 shows that the variance process exhibits correlation at a higher level than the raw returns. This is consistent with the setting of an ARCH type model. Note that the ACF shown in this figure appears to die out slowly, indicating the possibility of a variance process close to being nonstationary so we expect a high persistence of the model.

3.4 Quantifying the correlation

The preceding qualitative checks can be quantified for correlation using formal hypothesis tests, such as the Ljung-Box-Pierce Q-test. We implemented the Ljung-Box-Pierce Q-test for a departure from randomness based on the ACF of the data. The Q-test is most often used as a postestimation lack-of-fit test applied to the fitted innovations (i.e., residuals). Under the null hypothesis of no serial correlation, the Q-test statistic is asymptotically Chi-Square distributed.

The first output, H, is a Boolean decision flag. H = 0 implies that no significant correlation exists (i.e., do not reject the null hypothesis). H = 1 means that significant correlation exists (i.e., reject the null hypothesis). The remaining outputs are the P-value (pValue), the test statistic (Stat), and the critical value of the Chi-Square distribution (CriticalValue).

3.5 Ljung-Box-Pierce Q-Test

We can notice that the returns of the Romanian BET-C index are correlated up to 10, 15 and 20 lags. We could actually use a simple autoregressive model to forecast the movement of BET-C index in time for the specific period.

1.0000	0.0000	60.7373	18.3070
1.0000	0.0000	67.9978	24.9958
1.0000	0.0000	82.3635	31.4104

We also notice that there is significant serial correlation in the squared returns when you test them with the same inputs.

1.0000	0	137.3255	18.3070
1.0000	0	172.4888	24.9958
1.0000	0	193.0380	31.4104

4. PARAMETER ESTIMATION

This section continues the analysis begun in Preestimation Analysis. It estimates model parameters and then examines the estimated GARCH model.

4.1 Estimating the Model Parameters

The presence of heteroscedasticity, shown in the previous analysis, indicates that GARCH modeling is appropriate. For the E-Garch model previously described, the output provided by the Matlab package is:

Mean: ARMAX(0,1,0); Variance: EGARCH(1,1)
Conditional Probability Distribution: Gaussian
Number of Model Parameters Estimated: 6

Table 1

E-Garch estimated parameters

Parameter	Value	Standard Error	T Statistic
C	0.0017552	0.0003396	5.1686
MA(1)	0.18795	0.035061	5.3606
K	-0.46118	0.085248	-5.4099
GARCH(1)	0.94611	0.0095574	98.9928
ARCH(1)	0.38955	0.024847	15.6781
Leverage(1)	-0.025314	0.016725	-1.5136

If we substitute these estimates in the definition of the EGARCH model the estimation process implies that the model that best fits the observed data is

$$y_t = 0.0017552 + 0.18795\, y_{t-1} + \varepsilon_t \qquad (4)$$

$$\log \sigma_t^2 = -0.46118 + 0.94611 \log \sigma_{t-1}^2 + 0.38955 \left[\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - E\left\{ \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} \right\} \right] - 0.025314 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right)$$

(5)

4.2 Postestimation Analysis

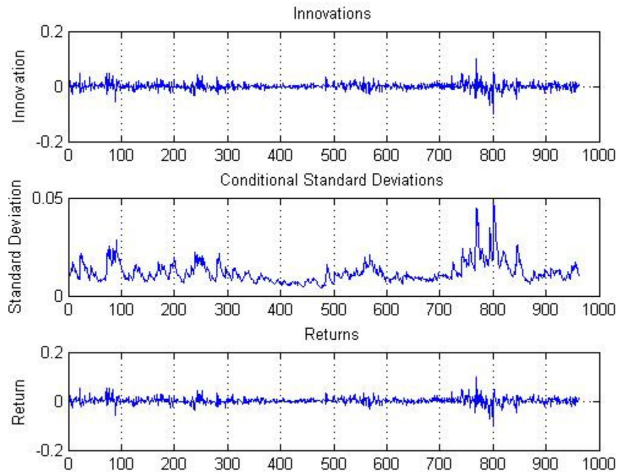
This part of the analysis starts by comparing the residuals, conditional standard deviations, and returns. It then uses plots and quantitative techniques to compare correlation of the standardized innovations.

4.1.1 Comparing the Residuals, Conditional Standard Deviations, and Returns

In addition to the parameter estimates and standard errors, we also computed the optimized log-likelihood function value (LLF), the residuals (innovations), and conditional standard deviations (sigmas). To inspect the relationship between the innovations (i.e., residuals) derived from the fitted model, the corresponding conditional standard deviations, and the observed returns are shown in Figure 5.

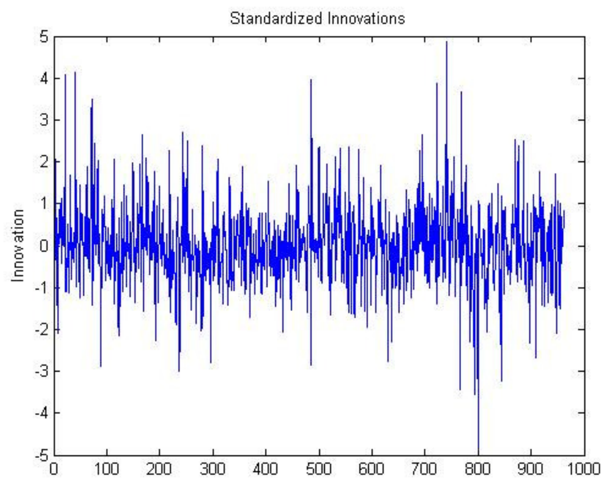
We can notice that both the innovations (top plot) and the returns (bottom plot) exhibit volatility clustering.

Figure 5 – Innovations, Conditional Standard Deviations and Returns



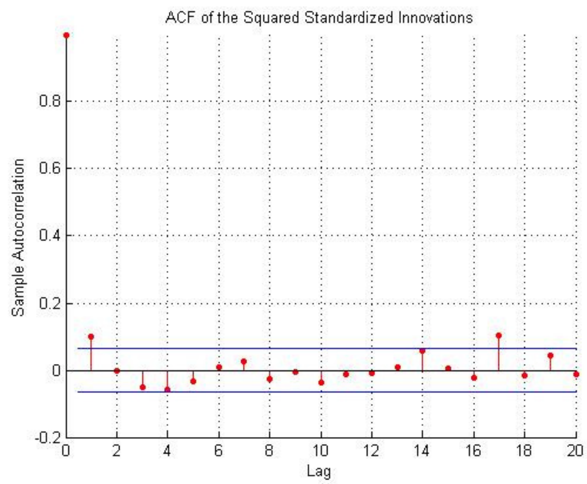
4.1.2 Plot and Compare the Correlation of the Standardized Innovations

Figure 6 – Standardized Innovations



If we plot the ACF of the squared standardized innovations, they show no correlation. This means that the E-Garch model did a good job by catching all the correlations in the returns – using the model as a filter we can see that the returns are random (the residuals behave as predicted by the model).

Figure 7 – Autocorrelation Function of the Squared Standardized Innovations



We can observe that by comparing the ACF of the squared standardized innovations in Figure 7 to the ACF of the squared returns prior to fitting the E-Garch, the model sufficiently explains the heteroscedasticity in the raw returns.

5. OUT OF SAMPLE TEST AND CONCLUSION

The parameters previously found were used in a mean squared error estimation of the parameters for E-Garch models in an out-of-sample test. We used 862 data from the 962 returns available from 3^{rd} of January 2002 until 17^{th} of November 2005 for the performance of this test.

The test is comprised of 100 one-day out-of-sample tests. For instance, we used the first 862 days to get the mean squared error estimate of the E-Garch parameters and then computed the one day ahead mean and standard deviation using this model. We checked if the actual return on that particular day (the 863^{rd} day of our sample) is inside a 95% confidence interval computed from the E-Garch estimation. We performed this test using a window of the same size (862 days) and testing the out-of-sample performance for all the 100 days.

We found that the out-of-sample returns were inside the E-Garch estimated 95% confidence interval for all the 100 days. We can actually use this model to perform capital market one-day estimates for the returns.

The next step in our research would be to use this procedure in order to check for similar results in individual securities.

References

1. Stulz, Rene M., *Rethinking Risk Management*, Journal of Applied Corporate Finance, Bank of America
2. Tsay, R. (2002), *Analysis of Financial Time Series*, Wiley Series in Probability and Statistics
3. Baillie, R.T., and T. Bollerslev, *Prediction in Dynamic Models with Time-Dependent Conditional Variances*, Journal of Econometrics, Vol. 52, 1992, pp 91-113
4. Bera, A.K., and H.L. Higgins, *A Survey of ARCH Models: Properties, Estimation and Testing*, Journal of Economic Surveys, Vol. 7, No. 4, 1993
5. Bollerslev, T., *A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return*, Review of Economics and Statistics, Vol. 69, 1987, pp 542-547
6. Bollerslev, T., *Generalized Autoregressive Conditional Heteroskedasticity*, Journal of Econometrics, Vol. 31, 1986, pp 307-327
7. Hamilton, J.D., *Time Series Analysis*, Princeton University Press, 1994

Radu LUPU, Lecturer PhD, Department of International Business and Economics, Academy of Economic Studies, Bucharest; Teacher at the Romanian Canadian MBA; Researcher at the Institute for Economic Forecasting, Romanian Academy, Bucharest. **Areas of expertise**: Risk Modeling, Asset Pricing, Econometrics. **Education and Training**: PhD in *International Business*, Magna Cum Laude, at The Academy Of Economics Studies in Bucharest, thesis: ”Risk Management And Financial Derivatives”; PhD candidate at the joint PhD program and researcher assistent at Concordia University, John Molson School of Business – Montreal. **Papers**: Author – ***Using BDS Test to Check the Forecasting Power of the Stochastic Volatility Model***, Economic Computation And Economic Cybernetics Studies And Research, June 2006, ISSN: 0424-267X, Academy of Economic Studies, Department of Cibernetics, Statistics and Informatics Economics; Author - ***The computation of bond values by simulating transition matrixes and calibration of BDT***, Economy Informatics, ISSN: 1453-1305, English issue on 2006; Author – ***Option bounds for multinomial stock returns in Jump-diffusion processes – a Monte Carlo simulation for a multi-jump process***, Romanian Journal of Economic Forecasting, Economic Forecasting Institute, ISSN: 1222-5436, 120, 2006 (forecaming issue); Co-author – Radu Lupu, Iulia Lupu – ***An Econometric Event Study for the Stocks listed at the Romanian Stock Exchange*** – Oeconomica, year XIV, no. 4, ISSN: 1223-0685

Iulia LUPU, Researcher at Victor Slavescu Center for Financial and Monetary Research, Romanian Academy. **Research Fields** - Capital Markets, Macroeconomic Analysis, Corporate Governance. **Education and Training**: Ph.D. Program at the National Institute for Research in Economics within the Romanian Academy; thesis theme: *Risk and Entropy on Capital Markets;* 2004 – 2005: one year PhD courses in Business Administration – common program: HEC Montréal (École des hautes études commerciales de Montréal), Concordia, McGill şi UQAM (Université du Québec à Montréal); research scholarship.