Solving the Capacity Optimization Problem under Demand Uncertainty

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This paper provides theoretical and practical insight into the solution of the investment project optimization problem under uncertainty. A case study recommends the use of statistical simulation, which is shown to be a powerful, practical, flexible and comprehensive tool for managerial decision-making purposes, even as it takes into account exogeneous uncertainty, as well as endogeneous processes structurally vested in the project. Results show that excess capacity may have a value exceeding its cost, which can be assessed either through comparison of available variants, or by carrying out a full optimization. From the theoretical point of view, the relationship of the problem to real-option analysis is investigated in more detail. Even though it obviously does contain real options, in principle, as various project alternatives differ in terms of their flexibility to alter operating scale, the proposed solution clearly surmounts some of the limitations of prevalent real-option models.

Key words: *capital budgeting, capacity optimization, real options, statistical simulation*
JEL classification: *C63, G31, M21*

1. Introduction

Establishing the optimal production capacity for a capital budgeting project is one of the essential, albeit most challenging, tasks faced by management. Besides the many inputs which ultimately relate to the technical specification of the project under consideration, it is therein critical to perform rather precise estimates of market demand, usually over a period comprising many years. The investment decision is then, to a large degree, irreversible, because subsequent project amendments usually incur prohibitive additional expenses, in line with the principle of sunk costs.

Any estimate of future demand is inherently uncertain, however, and the capital budgeting decision thus embraces substantial risk. On the one hand, ensuing from the commitment to carry fixed costs under a lower demand than formerly expected (operating leverage), on the other hand, from the potential lack of capacity to satisfy a demand in excess of the project's physical capacity constraint (opportunity cost). Complicating matters further, management often is in a position to, and should, actively react to current market development, typically through the use of diverse marketing tools.

The currently well-researched real-option methodology (for a review of its miscellaneous applications see e.g. Reuter and Tong, 2007) strives to quantify the value of project flexibility, which can be achieved by means of various technical and organizational features in a project. As classified by Trigeorgis (1993), is is thus possible to consider e.g. options to expand, which respond to an exogeneous demand increase, or options to contract, reacting to exogeneous drops in demand. On a related topic, Kulatilaka (1988) suggested using this method to assess the benefits of implementing flexible manufacturing systems (e.g. CNC machines). In his break-grounding papers, Myers (1977, 1984) stressed that the existence of such real options should properly be taken into account when assessing the value of any project.

On a rudimentary level, real options can be interpreted as a methodological analogy of financial options, which is the reason why many authors still tend to utilize somewhat misleading terms like "underlying asset", and highlight potential applications of closed-form formulas similar to the one designed by Black and Scholes (1973). Most efforts to resolve real-option problems through solving stochastic differential equations in real time have remained academic in nature, however, due to the inherent complexity of real systems, even though some of these generalizations have brought immensely valuable insights into essential management decisions, such as project deferral or staging, scrutinized by McDonald and Siegel (1986).

Most practitioners, on the other hand, give clear preference to methods, based on discrete numerical analysis. These can be divided into two basic categories. One is rooted in the simple and elegant backward-induction lattice model introduced by Cox, Ross and Rubinstein (1979), Rendleman and Barter (1979), and Sharpe (1978), which, in absolute terms, is currently the most popular method for evaluating miscellaneous types of options, including real options.

The alternative generic option-valuation method lies in the use of statistical simulation (Monte Carlo). This technique has been initially proposed by Boyle (1977), perhaps somewhat prematurely. Whereas it has not gained much acceptance for the valuation of financial options, with the exception of certain types of path-dependent exotic options (Wilmott, 2006), primarily because of high computing-power demands, it bears a strong potential for the solution of particular real-option problems. Its virtues are acknowledged wherever complex dynamic systems with various interdependencies defy conventional analytical tools. Since its introduction by Metropolis and Ulam (1949) and computerization in the Manhattan Project (Ulam, 1983), Monte Carlo has thus gradually become mainstream for applications in areas ranging from Physics to Technology to Natural Sciences (Fishman, 1996).

Most real-option theory and applications have featured options "on" projects, which readily submit to the financial-option analogy, typical examples being the development of land or other natural resources, as illustrated in the early contribution by Brennan and Schwartz (1985). Only recently has more focus been brought to options "in" projects (de Neufville, 2004, Wang, 2008). These may be harder to identify and involve complex operational relationships endogeneous to the project. Whereas earlier authorities (Dixit and Pindyck, 1994, Trigeorgis, 1996) tended to stress the nature of real options as rights or opportunities on projects, Luenberger (1998) already noted that it was possible to view virtually all processes allowing control as a series of operational options.

Besides, some surveys among major companies, such as those by Block (2007) or Razak and Kocaoglu (2001), indicate that actual usage of real-option methodology is much sparser than its glamorous image would seem to imply. All of these factors may have contributed to some practitioners' gradual diversion from the perception of the whole investment project (e.g. technological unit) as a black-box-like option-carrying entity, towards identifying particular flexibilities within the design of the system.

Such an approach effectively separates a particular course in real-option analysis from the contingent claims option-valuation mainstream, acknowledging its roots in operations research and engineering (Bellman, 1957, Charnes and Cooper, 1959), rather than in studies on economic equilibrium (Modigliani and Miller, 1958, Fama, 1970). This is, incidentally, evident when reviewing some pre-Black-Scholes capital budgeting applications, including Magee (1964), Robichek and Van Horne (1967) or Lewellen and Long (1972), and, significantly, Henry (1974), who had stipulated the essential irreversibility condition well before Myers (1977) even suggested the real option term and analogy. Some authors, including Smith and McCardle (1999), have actually raised the point of redundancy of the real-option paradigma in the face of existing decision-analysis methods.

This paper follows course to introduce statistical simulation as an optimization problem-solving tool in regard to the capacity design of a production facility. A case study demonstrates the strengths and particular characteristics of this method, facilitating its use as a corporate management heuristic. The vigilant reader may note the absence of the term "real option" in subsequent text, incidentally illustrating some lack of relevance when studying this kind of problem.

2. Defining the Problem for Conventional Analysis

Let us first consider a production-facility project budgeting an initial investment I and projected life T. The business plan establishes expected periodical sales of N units, assuming a unit price P, with direct unit costs U and periodical fixed costs F. Terminal (salvage) value is estimated at Z. The required return is stipulated to be r. For the sake of lucidity, we omit factors such as taxes and projected trends.

Conventional investment analysis of such a project is very simple, as demonstrated by any current business-school level corporate finance textbook, such as Emery, Finnerty, Stowe (2007). Namely, it suffices to calculate the Net Present Value (NPV) of the project

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} \qquad (1)$$

where CF_t denotes net cash flows (negative or positive) projected for each budgeted period, r the applicable discount rate. In case the project is independent, it should be accepted if $NPV > 0$. Of mutually exclusive project alternatives, the one with the highest NPV would be chosen.

For instance, estimates of I = \$90 million, T = 5, N = 100,000, P = \$1,000, U = \$600, F = \$5 million, Z = \$13.5 million, r = 12% result in NPV = \$43.8 million. The project would thus be readily accepted, unless the decision-maker is able to identify a mutually exclusive alternative project with a higher NPV.

However, this rudimentary and time-tested approach, first proposed by Fisher (1907), effectively ignores uncertainty of projected market demand on the one hand (not to speak about other risk factors, such as future market prices for the product), but also management capability to react to this major uncertainty on the other hand.

The optimum capacity problem thus reduces to the simple question whether marginal capacity incurs any marginal expenses; evidently, the discount rate has to remain unchanged, because production capacity has no relation whatsoever to systematic risk borne by the project. If that is the case, conventional investment analysis consistently recommends projects which exactly match the best estimate of demand, as alternatives with excess capacity offer a lower *NPV*.

In other words, using the conventional capital-budgeting toolkit, companies should normally not consider developing any capacity above mean expected demand, because that would imply higher project costs (typically, more machines would have to be purchased, plant space allocated, etc., to accommodate more capacity) and/or higher fixed expenses (e.g. insurance, maintanance), with no compensating benefits whatsoever.

To some degree, the deficiency can be mitigated by means of a scenario analysis. However this is still not sufficient to investigate the true nature of the various value drivers of a project, nor to resolve an optimization problem.

3. Defining the Problem for Statistical Simulation

Using statistical simulation, the same problem can be defined using much more realistic (and complex) assumptions, at the same time readily providing the decision-maker with quantitative results that can be used for subsequent optimization. To highlight the various qualities of Monte Carlo, we extend the former problem definition as follows:

First of all, it is necessary to select one or more basic risk factors. In a simple model, this can be just future market demand D_t, which determines the maximum amount that can be sold at then-current market terms (input costs could be another risk factor, under circumstances, particularly when involving a commodity). Demand will be defined as a stochastic process with parameters based on market analyses, and generated by computer for each project period and simulation step. The particular nature of this process will be stipulated later.

Actual production will also be determined by the maximum capacity N_{max} of the particular project variant being considered. Ignoring any changes in finished-goods inventory (lifting this constraint is viable, but requires a definition of inventory management rules for the model), the number of units produced and sold in a particular period N_t is defined as

$$N_t = \min\{D_t; N_{max}\} \quad (2)$$

Statistical simulation further allows setting algorithms to simulate the exogeneous or endogeneous impacts of changing demand.

Above all, it is realistic to assume that demand will have some influence on unit revenues of sales. Namely, low demand normally stimulates competitors to offer discounts or incentives to distributors, high demand diminishes the need for such incentives, and may even lead to upward price adjustments. An appropriate function $P_t = f(D_t)$ should thus be defined, either continuous or discrete.

This model includes a simple control mechanism of discrete price adjustments subject to some minimum change in demand compared to the previous period, as defined by the following function:

$$P_t \mid (D_t - D_{t-1} < D_{t-1}\ \delta_{neg}) = P_{t-1}\ (1 - \Delta_{neg})$$
$$P_t \mid (D_t - D_{t-1} > D_{t-1}\ \delta_{pos}) = P_{t-1}\ (1 + \Delta_{pos}) \quad (3)$$
$$P_t \mid (D_{t-1}\ \delta_{neg} \leq D_t - D_{t-1} \leq D_{t-1}\ \delta_{pos}) = P_{t-1}$$

The price-adjustment process is thus parametrized by δ_{neg}, δ_{pos}, Δ_{neg}, and Δ_{pos}, acknowledging that reactions to positive and negative demand fluctuations need not be symmetric, in reality. Note that (3) definitely does not represent a demand function, defining market equilibrium (Varian, 1992); it is rather a consequence of tactical decision-making by firms in the marketing process.

A particularly strong decline in demand may also be followed by a one-off demand-invigorating action, for example an extraordinary marketing campaign. This is defined by a simple discrete function (4), specifying an additional fixed expense *M*, to be incurred in period *t*, consequent on the fall of demand below a particular benchmark D_{mark} in the previous period.

$$M_t \mid (D_{t-1} < D_{mark}) = M \quad (4)$$

$$M_t \mid (D_{t-1} \geq D_{mark}) = 0$$

Finally, the model also perceives terminal project value Z_T to be contingent on demand. Salvage value is generally quite hard to estimate in capital budgeting projects, partly due to ambiguity as to whether the facility would be sold for scrap, transferred to second-hand buyers or generic producers, or perhaps retained for continued production. Clearly, sustainable actual utilization of the facility over a relatively brief period preceding project termination is of vital importance in this decision. Accordingly, the model assumes that there exists a terminal value estimate *Z*, related to expected demand *D*, and that actual terminal value Z_T retains the same proportion to the lesser of the last two periods' volumes of production. This relationship can be defined as

$$Z_T = \min\{N_{t-1}; N_t\} Z / D \text{ for } t = T \quad (5)$$

To facilitate comparison for various sizes of the project, we further state the terminal value estimate in terms of the residual value ratio of the initial investment, in line with common practice:

$$Z = \zeta \ I \quad (6)$$

Such a model, while being quite realistic, by the way illustrates the various kinds of algorithms available in a statistical simulation. Figure

2 (Appendix) illustrates the dynamic process, as it is handled by a single step of the simulation, in the form of a flow-chart diagram.

4. Case Study Parameters, Discussion, and Simulation Design

For case-study purposes we shall initially investigate a production facility project with two alternative specifications summarized in Table 1.

Table 1. Specifications of the alternative projects

Param.	Description	Alternative I	Alternative II
I	initial investment cost	\$90 million	\$93 million
T	project duration (periods)	5	
N_{max}	maximum production capacity	100,000 units	120,000 units
P_0	initial unit price	\$1,000	
U	unit cost	\$600	
F	fixed costs	\$5.0 million	\$5.5 million
ζ	residual value ratio ($Z = \zeta I$)	15%	
r	required return (periodic discount rate)	12%	

In the present case, project assessment periods are obviously assumed to be years (i.e. a 5-year project is being considered), but, generally, the budget could break down in shorter periods, i.e. quarters or months. This would be particularly relevant for projects which feature significant seasonalities, such as typically appear in e.g. agriculture, food processing, services and specialized manufacturing.

The model can also easily accomode trend expectations or irregularities in the projections of any chosen parameter, through either direct input of variables (the input matrix would thus expand to e.g. $\{U_1, U_2, \ldots, U_T\}$), or their initial values plus a periodic growth factor (i.e. using input of $\{U_0, \chi_U\}$).

Random exogeneous demand has been generated under the assumption of its normal (Gaussian) distribution around mean μ = 100,000 units, with a standard deviation σ = 15,000 units; this implies a perceived 68% probablility that periodic demand will remain within the range ⟨85,000; 115 ,000⟩ units.

Alternatively, a triangular distribution and BetaPERT distribution have been tested to assess distribution-driven sensitivity of the model. In both cases, parameters a = 63,000 units, b = 137,000 units, c = 100,000 units have been used, to achieve approximate calibration to the normal distrubution assumption in terms of mean and standard deviation.

The various parameters of the endogeneous controls have been summarized in Table 2.

Table 2. Control parameters

Triggering Parameter		**Value**	**Contingent Action**		**Value**
δ_{neg}	decrease in demand	-6.0 %	Δ_{neg}	unit price drop	5.0%
δ_{pos}	increase in demand	6.0 %	Δ_{pos}	unit price hike	2.0%
D_{mark}	low demand floor	90,000 units	M	campaign	$500,000

All simulations have been performed using the Crystal Ball simulation utility (for details on its characteristics see e.g. Charnes, 2007), with 100,000 experimental runs. This provides essentially real-time processing on readily available average-quality PC hardware, as well as a margin of error in *NPV* calculations under 0.4% (with 99% confidence), which seems to be perfectly acceptable for most corporate finance applications (median *NPV* results should thus be interpreted as e.g. $24.6 ±0.1 million).

5. Analysis Results

Running the simulation for the two basic project alternatives assuming Gaussian demand provides results summarized in the top part of Table 3.

Table 3. Project comparison (normal distribution of D_t)

Param.	**Description**	**Alternative I**	**Alternative II**
N_{max}	maximum production capacity	100,000 units	120,000 units
E(*NPV*)	median *NPV*	$24.6 million	$27.4 million
$NPV_{95\%}$	5th percentile of *NPV* distribution	$3.0 million	$1.8 million
$NPV_{99\%}$	1st percentile of *NPV* distribution	- $5.1 million	- $7.4 million

source: Author

Based on plain comparison, it is obvious that Alternative II (excess capacity) should be preferred to Alternative I, due to its substantially higher expected *NPV*, even when taking its less favourable risk characteristics into account and recognizing there is a margin of error in the E(*NPV*) estimates of ca $0.1 million. Strictly, the decision would be based on second-order stochastic dominance (Hadar and Russell, 1969) of Alternative II over Alternative I.

Using the alternative distributions, both of which might be considered for the same case, clearly does not bring any additional insight into the analysis (compare the results in Table 4 to Table 3). For the sake of simplicity and efficiency, the Gaussian should probably be preferred, as triangular is obviously unrealistic in the limit constraint areas, whereas BetaPERT is appreciably more demanding in terms of computing power.

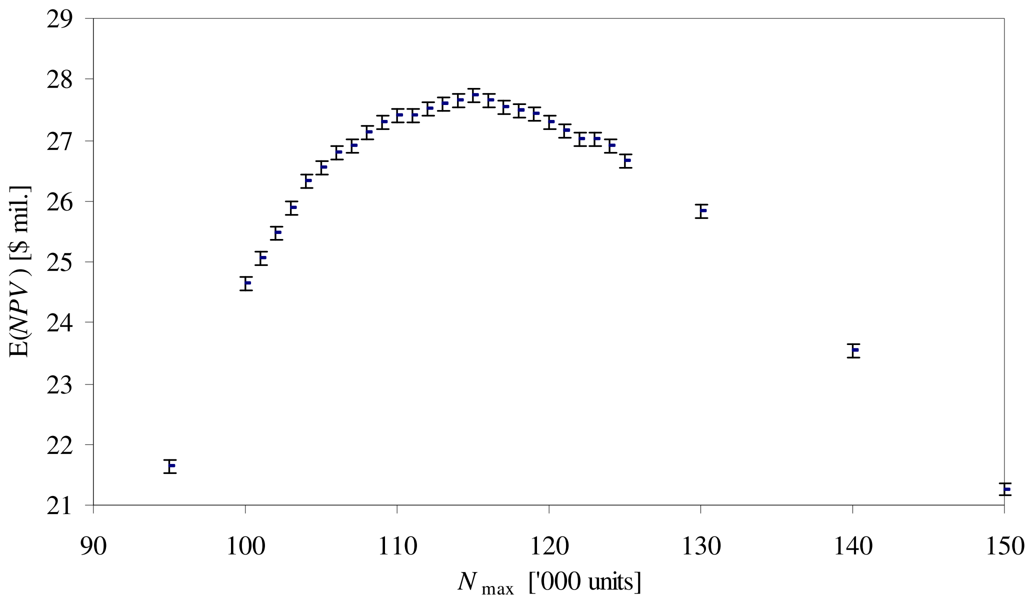
Table 3. Comparison under alternative distributions of D_t

Param.	Description	Alternative I	Alternative II
Triangular distribution			
E(*NPV*)	median *NPV*	\$24.0 million	\$27.1 million
$NPV_{95\%}$	5th percentile of *NPV* distribution	\$2.7 million	\$1.1 million
$NPV_{99\%}$	1st percentile of *NPV* distribution	- \$5.2 million	- \$7.9 million
BetaPERT distribution			
E(*NPV*)	median *NPV*	\$24.8 million	\$27.6 million
$NPV_{95\%}$	5th percentile of *NPV* distribution	\$4.1 million	\$2.6 million
$NPV_{99\%}$	1st percentile of *NPV* distribution	- \$3.4 million	- \$6.2 million

source: Author

Contrary to conventional scenario analysis, simulation results can also be used to perform full optimization of the $NPV(N_{max})$ relationship. Figure 1 illustrates that the function reaches optimum at $\hat{}N_{max}$ = 115,000 units with estimated ^E(*NPV*) = \$27.7 million.

Figure 1. Project optimization



source: Author

In practice, the investor can then make use of such an optimization to investigate whether another project alternative would be available, providing a maximum production capacity of ca 115,000 units.

6. Concluding Remarks

The case study shows that, contrary to conventional project analysis, reserve capacity may have a value exceeding its cost, and Monte Carlo can be readily used to determine its optimal design. The simulation can be performed very comprehensibly and with relatively modest means, which makes it particularly appealing for managerial communication and presentation. Even though the case has been tuned to represent production facility analysis, the same approach can be taken to solve capacity problems in other kinds of business, including transport, trade or services.

Many will appreciate that statistical simulation conspicuously avoids the daunting mathematics of contingent claims valuation (as well as that of dynamic programming, in fact), while addressing much more complex and realistic processes. This, incidentally, allows some further observations on the application of real-option theory.

Even though the basic specifications used in the case study for Alternative I (i.e. no reserve capacity, see Table 1) do not differ from those which have been used for the conventional analysis numerical example (see Part 2), expected *NPV* has been shown to be much lower therein (${}^{I}E(NPV)$ = \$24.6 million versus ${}^{conv}NPV$ = \$43.8 million). This seems conspicuously contrary to the expectations of mainstream real-options theory, which, since Myers (1977), typically maintains that real options should have a strictly positive impact on *NPV*, as indicated by the guiding principle

$$Expanded\ NPV = Static\ NPV + Option\ Premium \quad (7)$$

quoted e.g. in Trigeorgis (1996, p. 124), or widespread statements such as "Real options are important in strategic and financial analysis because traditional valuation tools such as NPV ignore the value of flexibility" (Leslie and Michaels, 1997).

On the contrary, the case suggests possible dismissal of some projects that would otherwise be accepted if conventional criteria were used. Specifically, any proposal to build a capacity of 100,000 units requiring initial investment in the range I = (\$114.6; \$133.8) million would be impacted. We note that this finding is essentially commensurate with the conclusions made by Bertola (1988), as well as Pindyck (1988), each of whom has used different analytical resources.

It is also remarkable that the model-generated *NPV* distribution, shown in Figure 3 (Appendix), definitely does not resemble Gaussian, its best fit being a left-skewed beta distribution. This contrasts with the common real-option wisdom that *NPV* either follows a random walk, as in McDonald and Siegel (1986), or that real options induce a right-skewed asymmetry, as in Trigeorgis (1996, p. 123).

All of these observations ultimately lead to the conclusion that instead of perceiving flexibility (i.e. options, if that is the preferred term) as a value-bolstering factor, it may be more appropriate to acknowledge that it is rather the lack of flexibility in a design that diminishes its value.

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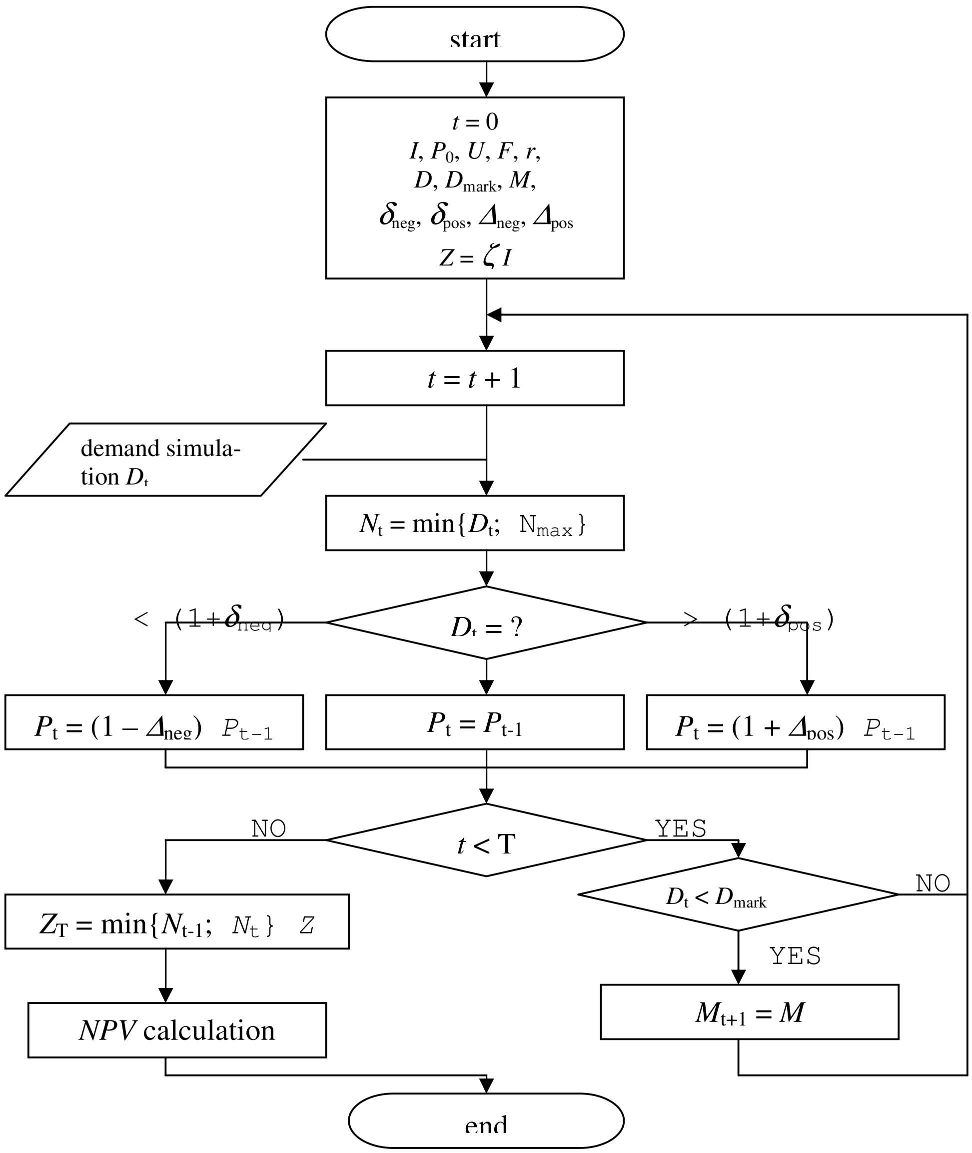
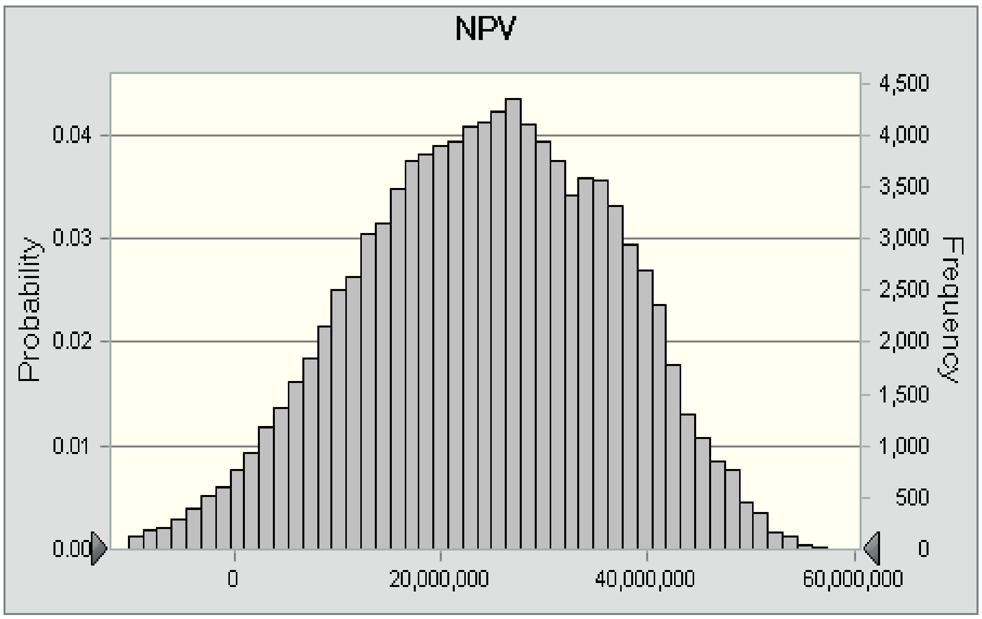
Appendix

Figure 2. Flow-chart diagram for a single simulation step of the model



Source: Author

Figure 3. Experimental *NPV* density function (N_{max} = 100,000 units, normally distributed D_t)



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